

MULTIDIMENSIONAL SCALING
OF ECONOMISTS' PERCEPTIONS OF ECONOMIC
SUBJECTS - AN INVESTIGATION,
INTERPRETATION, AND ANALYSIS

Charles Daniel Gee

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THESIS

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ECONOMISTS' PERCEPTIONS OF ECONOMIC
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by

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June 1975

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Multidimensional Scaling of
Economists' Perceptions of Economic Subjects -
An Investigation, Interpretation, and Analysis

by

Charles Daniel Gee
Lieutenant Commander, United States Navy
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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MANAGEMENT

ABSTRACT

The theory of multidimensional scaling is explained and the operation of a multidimensional scaling program (KYST) is examined under different input data forms and program control constraints. Using data collected from economists on the faculty of the Naval Postgraduate School, individual and aggregate configurations of their perceptions of economic subjects are obtained through multidimensional scaling of pairwise comparison data and are interpreted. A theory of non-interpretable dimensions is developed, and an application of multidimensional scaling as a technique for performance evaluation is suggested.

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I. INTRODUCTION

Multidimensional scaling is a relatively new computer-dependent technique for analyzing data of the type generally collected in the social and behavioral sciences. It has the unique ability to capture the complexity of the respondent's perceptions of the data and to portray these relationships within a two-, three-, or higher-dimensional space as appropriate to the respondent's perceptions.

Multidimensional scaling techniques are based upon well-established and clearly proved mathematical concepts which have been accepted since the mid-fifties; however, effective scaling programs and detailed studies of their operation under different input parameters and data constraints have appeared in the literature primarily in this decade. Although a large body of knowledge is rapidly being generated in this area, there are still many questions regarding the output of the program under various input parameters to which the answers are not entirely clear or which have not been intensively investigated. Thus an important purpose of this study was to investigate certain of these parameters by using data in which the final spatial configurations could be predicted in advance as a means of observing their effect.

An additional purpose was to investigate the operation of the program with actual data concerning economists'

perceptions of economic subjects. Questions addressed in this phase of the study included interpretation of results and the description, if possible, of the commonly held perceptions of economics by the respondents, all of whom were teaching at the Naval Postgraduate School. Finally, the meaning of non-interpretable configurations was analyzed, and a theory was developed to account for their existence. In addition, a potentially useful application of multidimensional scaling in performance evaluation will be proposed.

This study is organized into four main parts, corresponding to Sections II, III, IV, and V. The first section is expository, and explains the theory and operation of multidimensional scaling. The second (Section III) investigates technical aspects of the program's operation. Section IV provides an interpretation of the results of scaling individual and aggregate data, and Section V investigates the meaning of non-interpretable configurations.

The program used in this study was KYST (for Kruskal, Young, Shepard, and Torgerson, four of the leading pioneers in the field of multidimensional scaling). KYST represents a merger of the previously commonly used scaling programs M-D-SCAL 5M and TORSCA 9, and incorporates the best features of both. Since many of the studies in the literature are based on M-D-SCAL 5M, they may often be taken as representative of the operation of KYST except when they examine features which have been modified in KYST.

II. MULTIDIMENSIONAL SCALING - AN EXPLANATION

The purpose of this section is to acquaint the reader with multidimensional scaling by using an easily conceptualized model to describe the operation and theory of the scaling algorithm.

A. A DEMONSTRATION OF MULTIDIMENSIONAL SCALING

Although there are various computer-based techniques presently used for multidimensional scaling, all share a common purpose: to extract whatever pattern or structure which is otherwise hidden in a matrix of empirical data and to represent that structure in a form that is more readily accessible to the human eye, that is, through a geometric model or picture [Shepard, 1972].

For example, the mileage chart printed on many highway maps is a matrix of empirical data. Studying the mileage chart by itself would certainly not give the user any clear idea of the picture (in this case, the road map) from which it was derived. However, if the data from the mileage chart were used as input to a multidimensional scaling computer program, it should be able to extract the underlying pattern from which it was constructed--a map of the cities.

To demonstrate this basic property, the mileages from the matrix in Figure 1 were input to one of the most widely used multidimensional scaling programs, KYST.

Albuquerque, N. Mex.	273	1272	744	1138	1921	1350	187	1539	1457	490	182	820	721	1235	851	740	551	1551	1700	1159	613	4027	1360	1605	807	1037	1251	818	1350	7590	826	1317	1255	1123	943	1359	188	1282	271	12300	483	2540	609	1038	1099	1760	2493	2430	1368	393	424
Albuquerque, N. Mex.	273	1272	744	1138	1921	1350	187	1539	1457	490	182	820	721	1235	851	740	551	1551	1700	1159	613	4027	1360	1605	807	1037	1251	818	1350	7590	826	1317	1255	1123	943	1359	188	1282	271	12300	483	2540	609	1038	1099	1760	2493	2430	1368	393	424
Albuquerque, N. Mex.	273	1272	744	1138	1921	1350	187	1539	1457	490	182	820	721	1235	851	740	551	1551	1700	1159	613	4027	1360	1605	807	1037	1251	818	1350	7590	826	1317	1255	1123	943	1359	188	1282	271	12300	483	2540	609	1038	1099	1760	2493	2430	1368	393	424
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Figure 1

Great Circle Distances between Selected U.S. Cities in Statute Miles

Since the portion of the mileage chart above the main diagonal duplicates the data on the portion below the diagonal, it is only necessary to use one half of the matrix. Also, in order to check the output more accurately with a map, great circle distances, rather than road miles, are used as input data.

The result of putting the mileage matrix into the multidimensional scaling program is shown in Figure 2. The resulting picture seems to be a very good representation of the actual location of the cities. Figure 3 shows just how effective the program was in capturing the underlying configuration by comparing the location of the cities as determined by the program to their actual location on a U.S. map.

This simple illustration is an example of what Shepard [1972] calls the "classical" approach to multidimensional scaling. The input data were actual distances in miles; thus the actual difference in the distance between any two sets of points could be computed and was a meaningful number. To make the problem more difficult and to illustrate the ability of the newer "nonmetric" forms of multidimensional scaling, we will sort the $1/2(n)(n-1)=1225$ cells in the mileage matrix in Figure 1 into ascending order. Then we will assign a number, starting with the number 1 and incrementing by one for each distance that is larger than the previous one, to each cell in the matrix. When we finish, the values in each cell will indicate

CONFIGURATION PLOT DIMENSION 2 (Y-AXIS) VS. DIMENSION 1 (X-AXIS)
US GREAT CIRCLE DISTANCE ANALYSIS

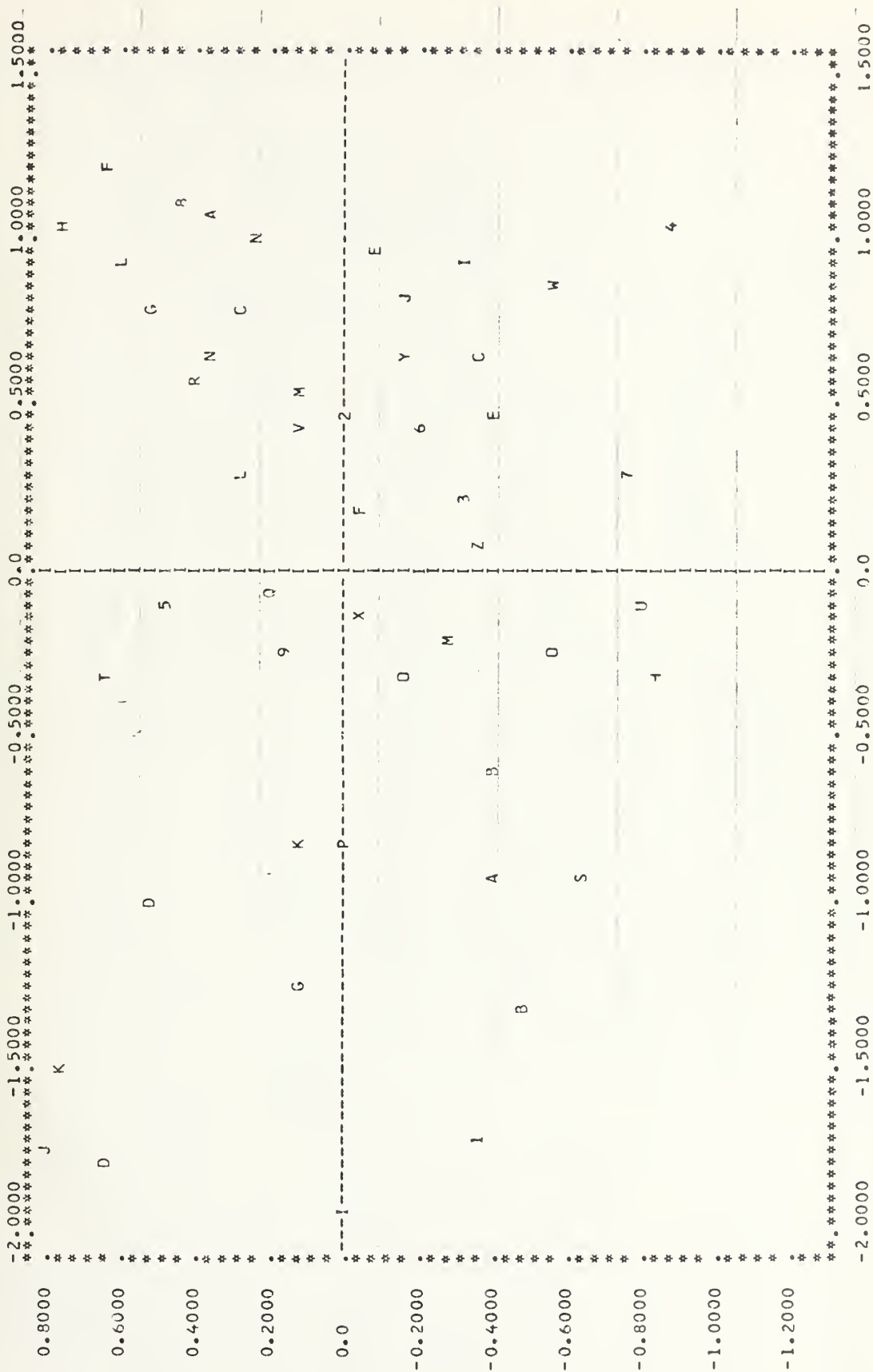


Figure 2

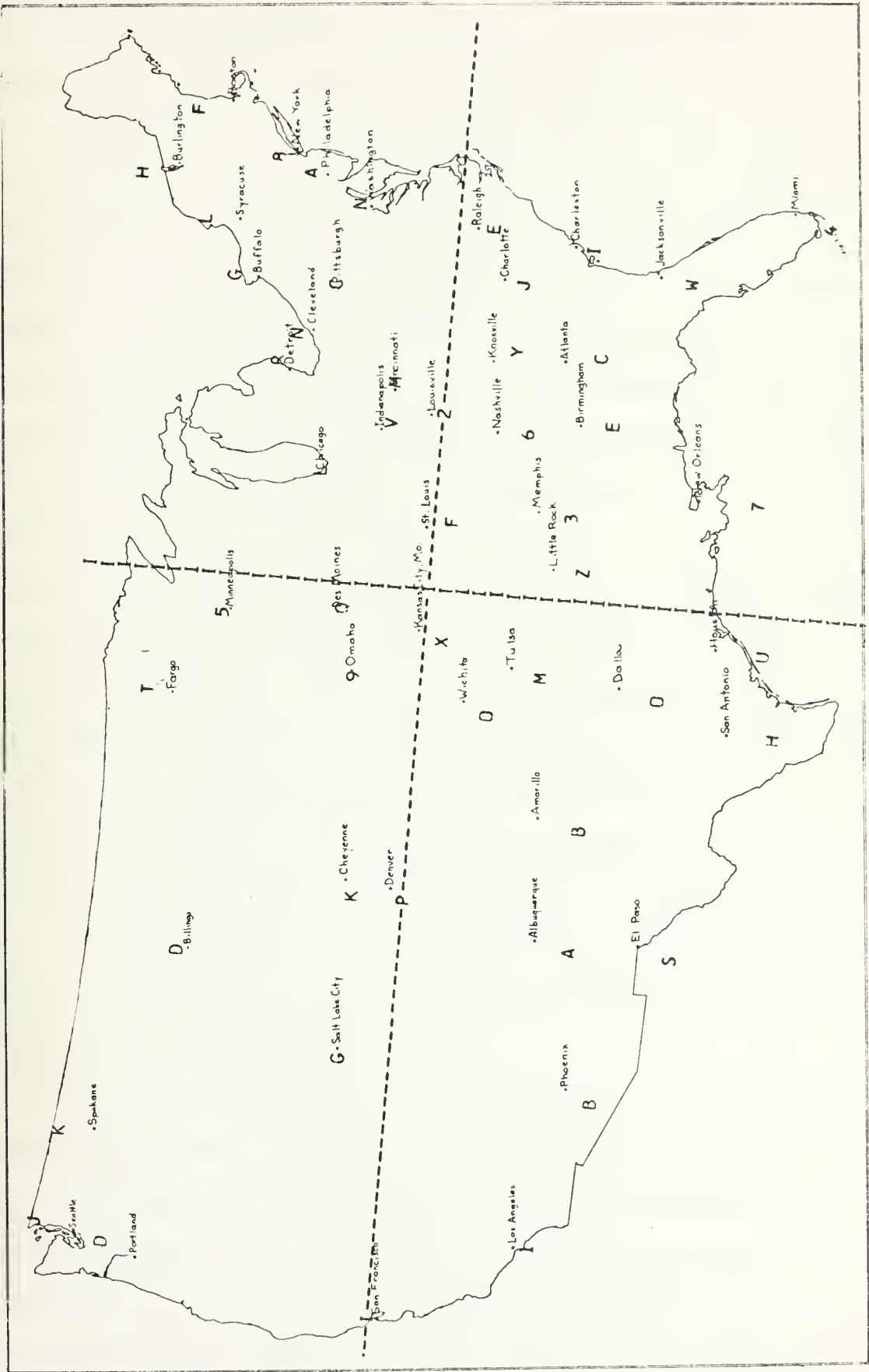


Figure 3

only whether one distance is larger or smaller than another, but not the amount of the difference.

From our new matrix we can determine the rank order of all possible pairs of cities, but we cannot say anything about the actual distances between cities with different rank orders. The newer varieties of multidimensional scaling, however, employ a methodology which seeks to discover (in our example) the actual distance relationships between all pairs of points, although this information is not directly apparent from the input data. By using this rank order matrix data as input for such a program, we obtain the configuration shown in Figure 4. Although this configuration is a mirror image of the actual map from which it is derived, the essential relationships between all points on the map are unchanged, as can be observed from Figure 5, which compares the output (after reflection and rotation by eye to the best orientation) to the actual location of the cities. From Figure 5 it is apparent that the scaling program has recovered the underlying configuration quite well, even though the input data only specified rank-ordering among the city-pairs.

This seemingly paradoxical ability to provide measurable output from rank ordered input is one of the most appealing and useful features of the newer varieties of multidimensional scaling. The operational algorithm upon which it is based is relatively simple and will be described on an intuitive level shortly. First, however, it will be

CONFIGURATION PLOT DIMENSION 2 (Y-AXIS) VS. DIMENSION 1 (X-AXIS) U.S. GREAT CIRCLE ORDINALLY RANKED DISTANCE ANALYSIS

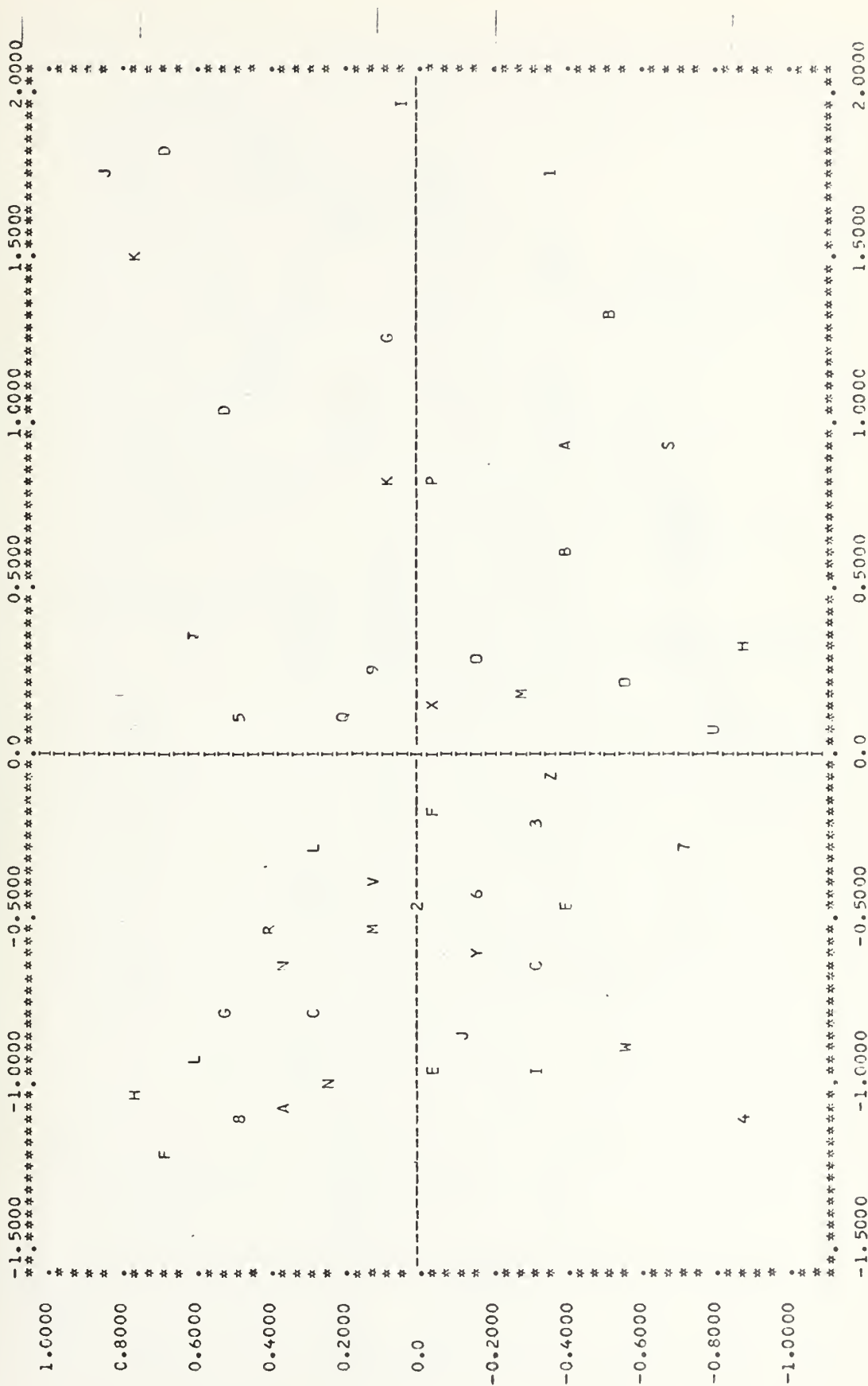


Figure 4

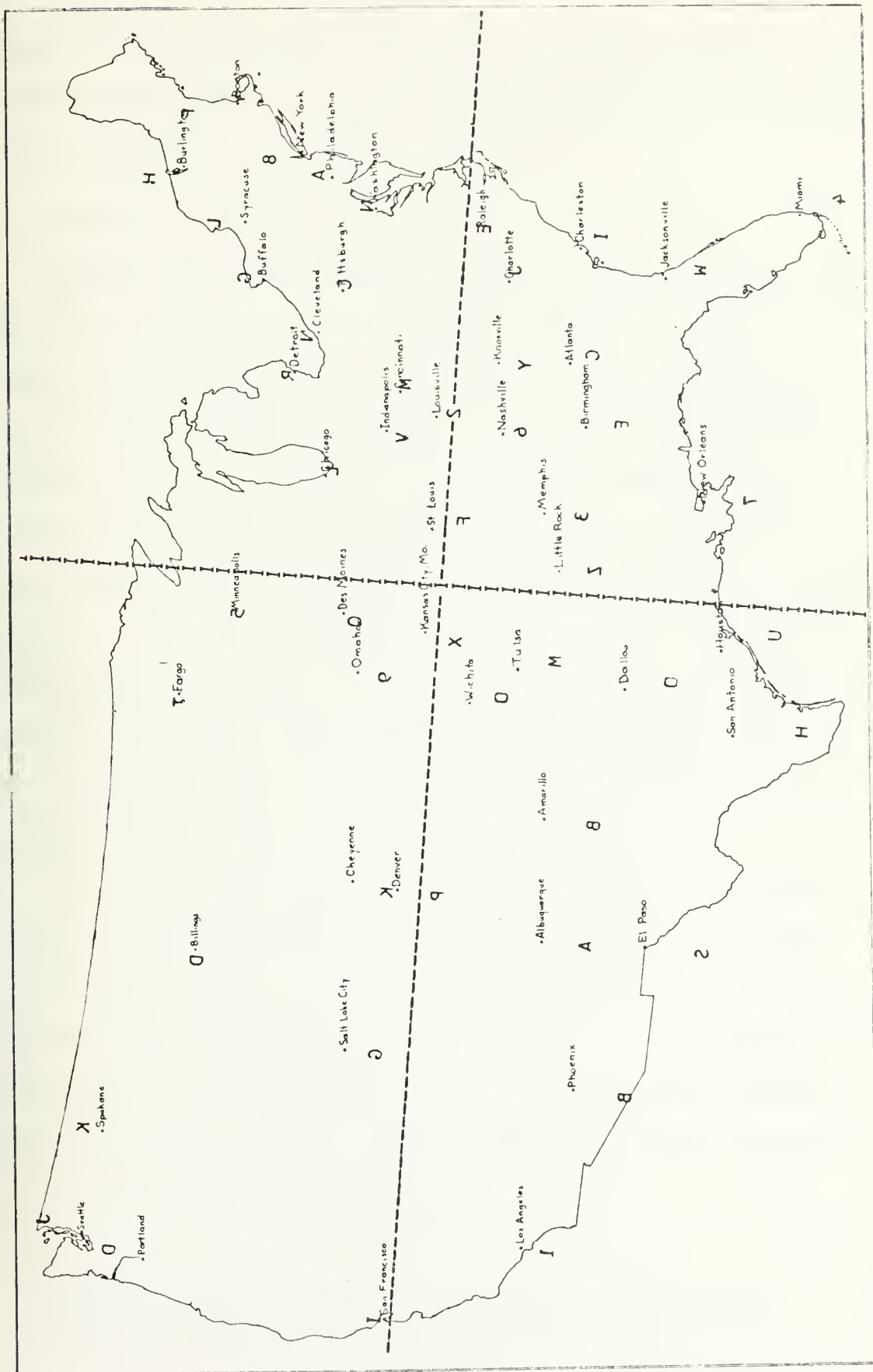


Figure 5

necessary to understand some basic principles germane to multidimensional scaling in order to explain the algorithm more precisely.

B. TYPES OF SCALES*

When we assigned numbers starting with 1 and increasing by one for each intercity distance that was greater than the previous one, we were developing a type of ordinal scale. We could have numbered them starting with 563, for example, just as well, for the numbers themselves did not denote distances; they only provided the rank order of the distances. Figure 6 depicts the unique property of ordinal scales, that of monotonicity or rank order. The numbers comprising an ordinal scale may be changed at will, but as long as their rank order (monotonicity) is preserved, they will provide just as good a representation of the empirical relationships as the original scale.

The left panel of Figure 6 illustrates an ascending monotone ordinal scale such as we used when we assigned increasing numbers to the intercity distances. If we assume that the Y-axis represents the actual distances and the X-axis the ordinal distances, any rank-order number x_2 which is greater than x_1 implies that the actual distance y_2 is greater than distance y_1 .

*This section, and the one following, are based primarily on Green & Carmone, 1970, pp. 7-24.

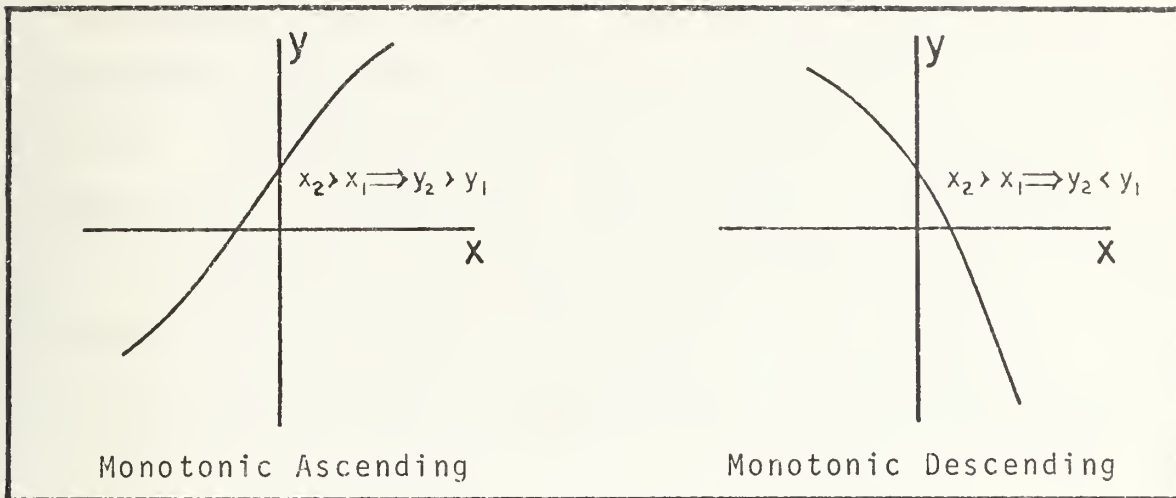


Figure 6

The right panel of Figure 6 illustrates a descending monotone ordinal scale. If we had started numbering the intercity distances with, say, 1,000 for the shortest and decremented the numbers until the largest distance had the lowest number, the monotone function would then be reversed; that is, any value of x_2 which was greater than x_1 would imply that the corresponding distance y_2 was less than y_1 . In connection with multidimensional scaling, ordinal scales are often referred to as "nonmetric," to distinguish them from the other two major types of "metric" scales, interval and ratio.

Interval scales assume an additional property not found in ordinal scales: the differences between the numbers themselves are meaningful and measurable. A common example of an interval scale is the one used to measure temperature, either Fahrenheit or centigrade. Figure 7 illustrates the essential function of interval scales, their linearity.

For example, converting temperature from Fahrenheit to centigrade represents a linear transformation of the form $y=a+bx$. In Figure 7, the X-axis could be considered as temperature in centigrade, and the Y-axis in Fahrenheit. It is not necessary that the line pass through the origin, however. This indicates that there is no requirement for a true zero or natural starting point for the measurement.

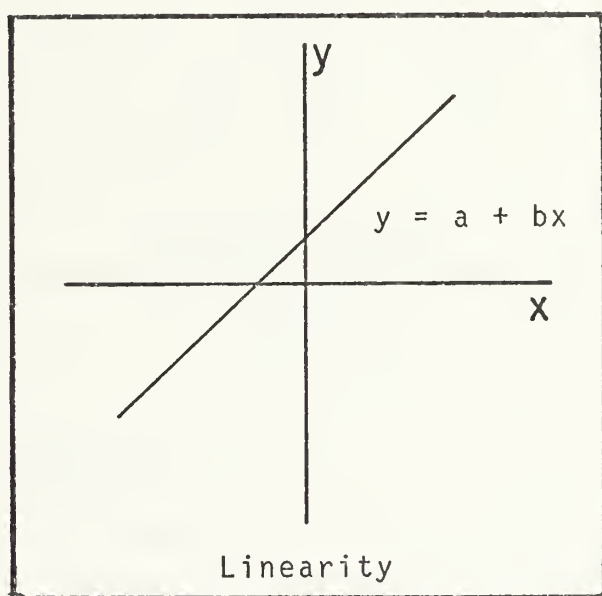


Figure 7

If a unique zero or natural origin point can be fixed, then we have a ratio scale. Our original mileage matrix was based on a ratio scale. Its unique quality derives from the fact that any unit of measurement on a ratio scale may be transformed into any other simply by multiplying by a constant. For example, we could have converted the mileage chart into a kilometer chart by multiplying

each entry by the appropriate conversion factor, .62.

Figure 8 illustrates this case.

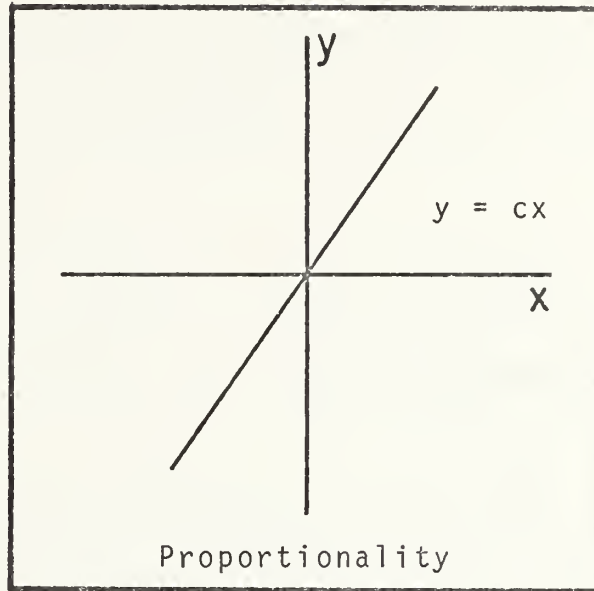


Figure 8

C. DISTANCE FUNCTIONS

In using our mileage chart or the ordinal distance chart to represent distances between all of the fifty city-pairs, we were assuming certain basic properties of the underlying configuration (the map itself). Specifically, we were assuming that the map was positioned in what is known as "metric space." A metric space is one which has a well-defined distance function possessing the following properties for any points x , y , and z :

1. The distance between a point and itself is zero, and the distance between distinct points is positive, i.e.,

$$d(x,x) = 0 \quad \text{and} \quad d(x,y) > 0$$

2. Distances must be symmetric, i.e.,

$$d(x,y) = d(y,x)$$

3. The distance from point x to point y must be less than or equal to the distance from x to y indirectly through point z, i.e.,

$$d(x,y) \leq d(x,z) + d(z,y)$$

This last distance function is known as the triangle inequality.

The Pythagorean theorem of high school geometry is based on these three properties. Thus to determine the distance between two points, x and y, in a two-dimensional metric space such as Figure 9, we only need to pick a third point z, such that a line from z to y is parallel to the X-axis (dimension 1), and a line from z to x is parallel to the Y-axis (dimension 2). By computing the difference between z and x, and squaring the result, and computing the difference between z and y and squaring the result, then summing the squares and taking the square root of the sum, we obtain the distance from x to y. In general, then, the distance d between two points i and j in the plane is given by:

$$d_{ij} = [(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2]^{1/2}$$

The above is known as the Euclidean distance function.

Conceptually it is a simple matter to generalize the Euclidean distance between two points into as many dimensions (from 1 to t) as we choose. In so doing the distance formula is more clearly shown as:

$$d_{ij} = \left[\sum_{k=1}^t (x_{ik} - x_{jk})^2 \right]^{1/2}$$

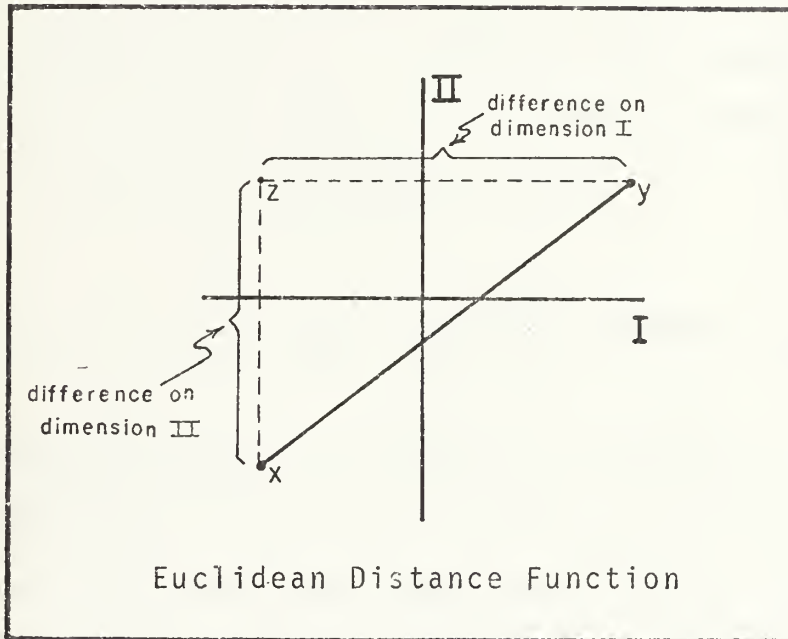


Figure 9

Finally, we can generalize the Euclidean distance function to any other type of distance. For example, instead of squaring the distances we could simply raise them to the power of 1. Because the absolute value of the distances is always positive in metric space, the result would be what is often referred to as "city-block metric." That is, to get from point x to y , one must always move

either north, south, east, or west. No diagonal moves are allowed. Thus, the city-block metric distance between any two points is the sum of the absolute difference of their projections on each separate dimension.

Likewise, we could imagine many other possible powers in our general distance function, providing that they were equal to or greater than 1 (so as not to violate the triangle inequality). In so doing, we have generalized the Pythagorean theorem into what is known as the Minkowski ρ -metric which is written as follows:

$$d_{ij}(\rho) = \left[\sum_{k=1}^t |x_{ik} - x_{jk}|^{\rho} \right]^{\frac{1}{\rho}} ; \text{ subject to: } \rho \geq 1$$

Before leaving this section, it should be pointed out that although the Minkowski ρ -metric satisfies all of the properties of a distance function, only a geometrical configuration based on Euclidean distance can be rotated around its origin without altering the relationships between the interpoint distances.

D. THE MULTIDIMENSIONAL SCALING ALGORITHM

The term multidimensional scaling encompasses a wide variety of techniques. The focus of this paper is on nonmetric multidimensional scaling of the type developed by J. B. Kruskal (1964 a and b). It is nonmetric in the sense that it requires as input only ordinal data. In general terms, the objective of multidimensional scaling

can be stated as: given a rank order of input data (such as our ordinal distance chart), find the configuration of a set of points whose rank order of ratio-scaled distances--in a specified dimensionality--best reproduce the original rank order of the input data [Green and Carmone, 1970]. From a statistical point of view, it is analogous to a least-squares regression in that we start with a rank order of intercity distances and we wish to find the configuration whose interpoint distances fits them best [Kruskal, 1964].

To describe in general terms how a typical multidimensional scaling program works, we will use as input data a small matrix of only four cities and their $1/2(4 \times 3) = 6$ interpoint mileages in two dimensions. This is shown in Figure 10. It should be recognized that this is only an expository illustration and is not intended to represent a reasonable matrix size. In fact, any number of configurations which preserve the original rank order can always be found for n points in $n-1$ dimensions; thus we would not expect the program to successfully recover the actual "map" from so few points.

		CITIES			
		1	2	3	4
CITIES	1	-			
	2	851	-		
	3	2596	1745	-	
	4	1255	1188	2339	-
		INTERPOINT MILEAGES (s_{ij})			

KEY:

1 = Boston
2 = Chicago
3 = Los Angeles
4 = Miami

Figure 10

These mileages can then be ranked to provide the following (ascending) order; where s_{ij} represents the mileage between cities i and j :

$$s_{21} < s_{42} < s_{41} < s_{32} < s_{43} < s_{31}$$

An arbitrary starting configuration of four points representing the cities would then be established by the program. This starting configuration may be completely random or determined by some other means. The specific details need not concern us here. Figure 11 shows one possible starting configuration.

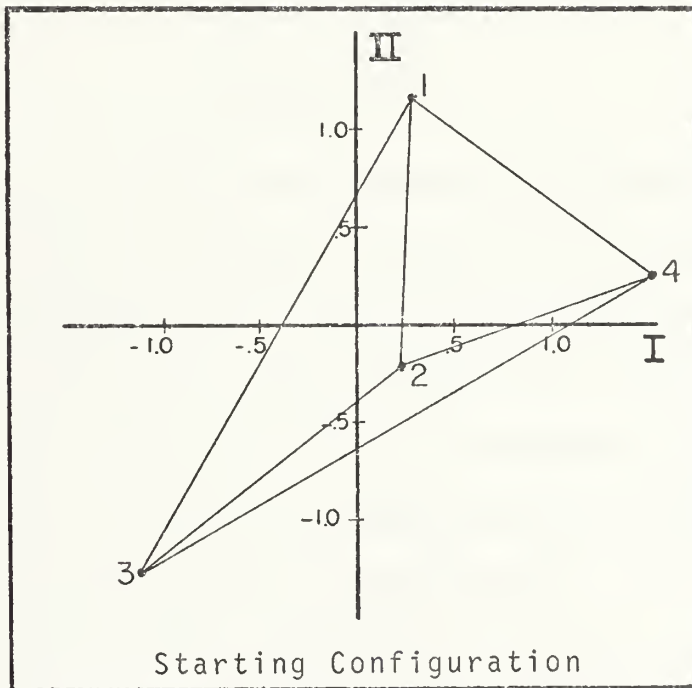


Figure 11

If we specify the use of the Euclidean distance function ($\rho=2$), the interpoint distances (d_{ij}) of the configuration would then be computed by the Pythagorean theorem which turns out for this example to be as shown in Figure 12.

		CITIES				KEY:
		1	2	3	4	
CITIES	1	-				1 = Boston
	2	1.3	-			2 = Chicago
	3	2.7	1.7	-		3 = Los Angeles
	4	1.5	1.2	2.9	-	4 = Miami
INITIAL CONFIGURATION DISTANCES (d_{ij})						

Figure 12

A central goal of multidimensional scaling is that the rank order of the interpoint distances in the configuration should be monotonically related to the rank order of the interpoint measures of the original matrix [Shepard, 1962]. This monotonic relationship may be ascending, i.e., directly related, if the original data measures are dissimilarities such as our mileage chart; or descending, i.e., inversely related, if the original data are proximity measures (where larger rank order numbers indicate a closer proximity). Regardless of the relationship, to achieve the goal of monotonicity, there must be a consistent relationship between the rank orders of the original data and the rank

orders of the distance functions. For rankings of original data based on dissimilarities this rule can be generalized as:

$$d_{ij} < d_{kl} \quad \text{whenever} \quad s_{ij} < s_{kl}$$

By ranking the d_{ij} 's from the initial configuration we obtain:

$$d_{42} < d_{21} < d_{41} < d_{32} < d_{31} < d_{43}$$

Comparing this with our original mileage ranking, where

$$s_{21} < s_{42} < s_{41} < s_{32} < s_{43} < s_{31}$$

we observe that our goal of monotonicity has not been achieved by the original configuration. This is, in fact, the usual condition when using real data in the program.

In order to determine a set of numbers which do satisfy the rank order constraint of our original s_{ij} 's we can take the arithmetic mean of the groups of d_{ij} 's which do not satisfy the rank ordering of the corresponding s_{ij} 's. These "average distances" will be referred to as \hat{d}_{ij} .

$$1/2(d_{42} + d_{21}) = \hat{d}_{42} = \hat{d}_{21} = 1.25$$

$$d_{41} = \hat{d}_{41} = 1.50$$

$$d_{32} = \hat{d}_{32} = 1.70$$

$$1/2(d_{31} + d_{43}) = \hat{d}_{31} = \hat{d}_{43} = 2.80$$

This example demonstrates that the \hat{d}_{ij} 's are derived by averaging certain contiguous groups of d_{ij} 's [Capra, 1970]. By determining the values of the \hat{d}_{ij} 's in this manner, we ensure that the resulting rank ordering is monotonically related to the ranking of the d_{ij} 's. That is,

$$\hat{d}_{ij} \leq \hat{d}_{k\ell} \quad \text{whenever} \quad d_{ij} < d_{k\ell} .$$

The computation of the \hat{d}_{ij} 's and their relationship to the configuration distances (d_{ij} 's) and the original data distances (s_{ij} 's) is at the heart of multidimensional scaling. Restating the preceding discussion through an illustrative example may assist in better understanding the concept. -

Returning for a moment to our initial configuration (Figure 11) our problem is to determine how well this configuration represents the data. Later on we will want to find out what configuration represents the data best. For the moment we are only concerned with developing the criteria by which to judge configurations.

Recall that our goal is to achieve a monotonic relationship between the interpoint distances in the configuration and those of the original data. Thus we can construct a scatter diagram, such as Figure 13, in order to graphically portray how well the configuration interpoint distances (measured on the X-axis) match the original input data mileages (on the Y-axis).

Using the two matrices in Figures 10 and 12, we can plot this relationship in Figure 13. Each point refers to a pair of cities, as shown.

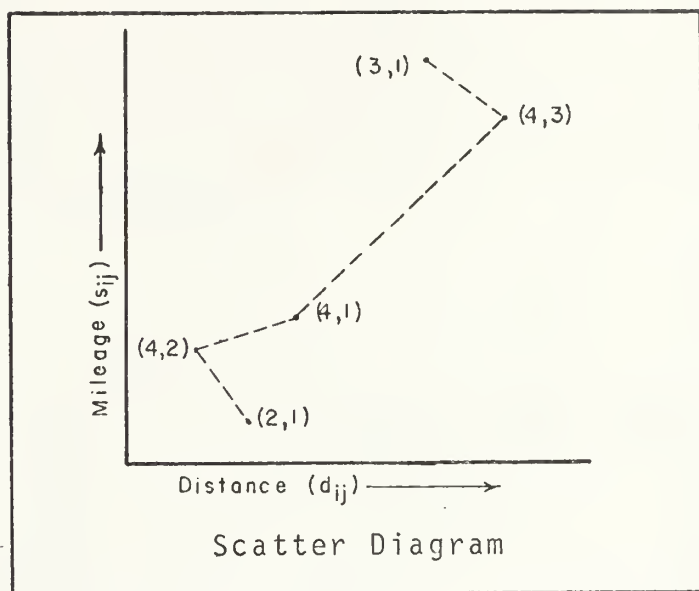


Figure 13

If our configuration is monotone with the mileage matrix, increases in distance will match increases in mileage, from smallest to largest. This means that if we trace out the points in Figure 13, one by one, from bottom to top, we will always move to the right, never to the left, so that the smallest distance comes first, then the next smallest, and so on. The dashed line in Figure 13 shows the results of tracing out these points. Because the trace goes from right to left between the first two points and the last two points, we can observe that our goal of monotonicity has not been achieved.

We can achieve the desired monotonic relationship by horizontally shifting the points which are not properly rank ordered to new points, represented by the \hat{d}_{ij} values we calculated earlier. In other words, we will fit an ascending curve to the data. Figure 14 shows the results after the curve has been so established. The dashed line is the original trace; the solid line the curve which has been fitted to the points.

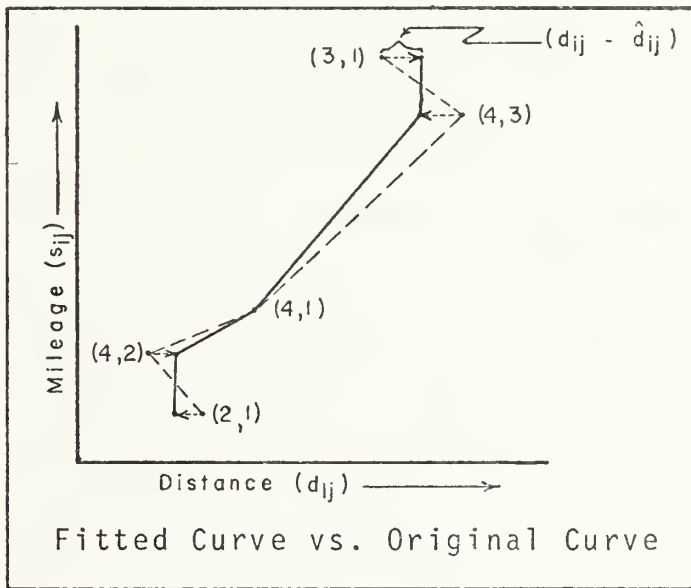


Figure 14

From Figure 14, we can intuitively understand that the differences between the \hat{d}_{ij} 's and their corresponding d_{ij} 's represent the minimum differences between the d_{ij} 's and any set of numbers which corresponds to the original rank ordering of the input data (the s_{ij} 's).

It should be recognized that the \hat{d}_{ij} 's and d_{ij} 's are measurements on a ratio scale. The s_{ij} 's, on the other hand, need only be ordinal measures, since the "curve fitting" algorithm performs arithmetic only on the \hat{d}_{ij} 's and d_{ij} 's.

Now that we have computed the values of the \hat{d}_{ij} 's necessary to fit a curve to our original configuration data, it is necessary only to compute precisely how well the configuration fits the original input mileage data. This is done by squaring the difference between each d_{ij} and its respective \hat{d}_{ij} for all M cells in the matrix and summing the squares. The result is divided by the summation of all d_{ij} 's squared to normalize the result, that is, make it invariant under changes of scale such as stretching or shrinking. The square root of the entire expression is then taken which is analogous to obtaining the standard deviation. Kruskal [1964 a) calls the resulting quantity "stress," a measure of how well the configuration represents the data. The stress formula is:

$$S = \left(\frac{\sum_{\substack{i > j \\ j=1 \\ M}} (d_{ij} - \hat{d}_{ij})^2}{\sum_{\substack{i > j \\ j=1 \\ M}} d_{ij}^2} \right)^{1/2}$$

If an upperhalf matrix were used as input data the constraining $i > j$ would be changed to $i < j$.

The value of "stress" is analogous to the coefficient of correlation, except that a large value of stress indicates a bad fit, while a small value indicates a good fit. Now that stress has been defined, the problem which a multi-dimensional scaling program seeks to solve can be stated as: minimize stress, subject to the requirement that each \hat{d}_{ij} must be greater than or equal to the one before it. Stated algebraically, this is:

$$\begin{aligned} &\text{minimize} \left(\frac{\sum_{\substack{i>j \\ j=1}}^M (d_{ij} - \hat{d}_{ij})^2}{\sum_{\substack{i>j \\ j=1}}^M d_{ij}^2} \right)^{1/2} \\ &\text{s.t.: } \hat{d}_{i_1 j_1} \leq \hat{d}_{i_2 j_2} \leq \dots \leq \hat{d}_{i_M j_M} \end{aligned}$$

The minimum stress configuration is found by moving all of the points in the original configuration just a little, recalculating the d_{ij} 's and \hat{d}_{ij} 's and the stress, and comparing the new stress value with the previous value. If the new stress value is lower, all the points are again moved a little, and the stress recomputed. This iterative process continues until either a preselected minimum stress value (usually .01) is achieved, or the percentage improvement in stress from one iteration to the next becomes negligible, or a preselected maximum number of iterations (usually 50) have been performed.

This iterative stress procedure is performed for as many dimensions as are requested, generally up to a maximum of six. Minimum stress is first computed for the maximum number of dimensions specified, then for the maximum number minus one, and continues to finally, a computation for only one dimension. Thus, in our example, minimum stress would have first been computed for the configuration in two dimensions, then a one-dimensional (straight line) configuration would have been established and the iteration begun again to compute minimum stress in one dimension.

E. DETERMINING AND NAMING THE DIMENSIONS

Although multidimensional scaling programs will provide configurations in several dimensions, it is the analyst's task to (1) determine the appropriate number of dimensions and (2) provide names for these dimensions. Shepard [1972] suggests four criteria for determining the correct number of dimensions:

(1) The final stress value should not be too large, or should not drop too abruptly as further dimensions are added. A plot of stress versus dimensions, such as Figure 15, is provided by the program to assist the analyst here. By observing Figure 15, which is a fairly typical plot, we can see a clear "elbow" in the curve at three dimensions. Beyond this point, adding more dimensions does not significantly decrease the stress. Since the analyst generally desires to work with the minimum number of dimensions, in

this illustration he would confine his study to only the first three dimensions.

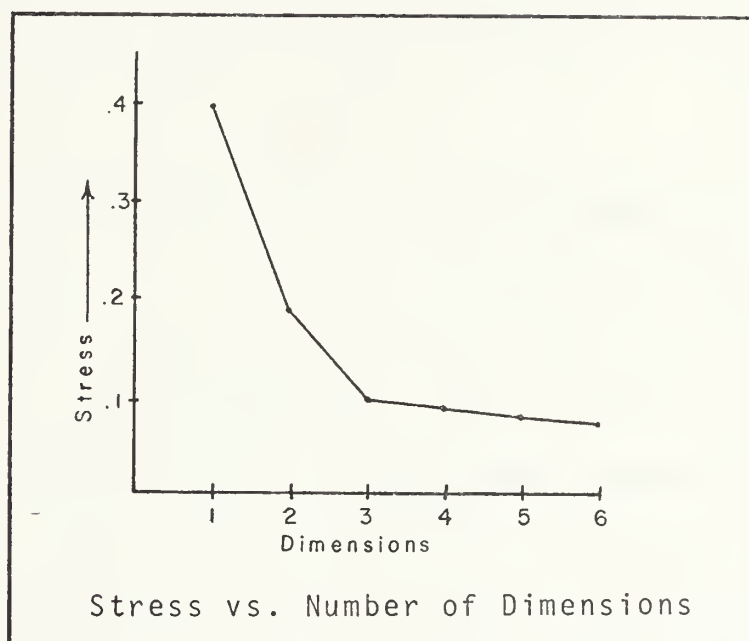


Figure 15

(2) The representation should be statistically reliable. That is, for solutions obtained separately for two independent sets of data, the final configurations produced should be in close agreement. Generally, when more dimensions are permitted than are necessary, this agreement will tend to break down.

(3) The representations should be interpretable. For our mileage map, this was simple in two dimensions. Dimension I was an east-west representation; and dimensions I and II provided an east-west, north-south representation.

For other types of data, however, the task of naming the dimensions may not be so simple and may require some subjective judgments on the analyst's part.

(4) Finally, Shepard suggests that, except for special types of multidimensional scaling programs, the number of dimensions not exceed three in order that the representation can be readily visualized.

F. DETERMINING THE CORRECT DISTANCE FUNCTION

Recalling the earlier discussion of the Minkowski ρ -metric distance function, it was indicated that any distance function equal to or greater than one could be used in multidimensional scaling. However, only the Euclidean function ($\rho=2$) allows rotation of the configuration around the axes while maintaining the correct interpoint distances. The analyst must determine which distance function properly represents the configuration underlying the data.

Sherman [1972] conducted extensive studies to determine the importance of accurately estimating the distance function. His findings indicate that if the attributes of the stimuli are perceptually distinct (as are height and tilt of parallelograms), the city-block metric ($\rho=1$) may lead to a better model. If the attributes of the stimuli interact (such as hue and brightness of color chips), greater values of ρ may lead to a better model. If there are numerous interacting attributes of stimuli and yet one appears to dominate the others for a particular stimulus when paired with another

stimulus, larger values of ρ , such as 8, 16 or 32 should be tried.

Cunningham and Shepard [1974] in discussing the question of the proper value of ρ state that "...accumulated experience indicates that unless the underlying structure has an extreme nondimensional form...the spatial configuration of the n objects with respect to the first two or three principal axes will generally be quite robust, i.e., insensitive to departures of the true underlying metric from the Euclidean."

We can perhaps summarize by observing that while the question of the correct value of ρ has still not been conclusively settled, we would likely be on solid grounds in using the Euclidean distance ($\rho=2$) unless there is reasonable doubt as to its applicability. If such doubt exists, iterative scalings with different ρ -values and interpretation of the results would likely suggest the best value to employ in a given problem.

G. USES OF MULTIDIMENSIONAL SCALING

Actual applications of multidimensional scaling usually involve presenting subjects with a list of all possible pairs of various stimuli and having the subjects rank each pair on some ordinal scale to denote their perceived similarity or dissimilarity. The resulting data are used to develop an input matrix, and the output represents a kind of "psychological map" of the subjects' perceptions

of the stimuli. Observing the pattern of the stimuli in the representation and determining the meaning of the various dimensions employed can often provide fresh insights into the subjects' perceptions of the stimuli.

At present, multidimensional scaling has been employed in disciplines such as psychology [Rosenberg and Sedlak, 1972]; marketing research [Green and Rao, 1972]; the electoral process [Mauser, 1972]; semantics [Rapoport and Fillenbaum, 1972]; and financial analysis [Krampf and Williams, 1974]. It appears to have potential value in other areas, including economics and the social sciences. A new potential application, performance evaluation of professional persons, will be discussed in this thesis.

III. METHODOLOGY, DATA COLLECTION, AND PROGRAM ANALYSIS

A. METHODOLOGY

This investigation into economists' perceptions of economics was conducted in two phases. The first phase was a "test" phase. Data collected from a small sample of economists were used to study the operation of the multi-dimensional scaling program using different forms of input data and different program control parameters. The information gained was then used to establish some tentative hypotheses concerning the effect of these variables on the final configurations obtained and to suggest the input data forms and program control parameters which should be used to obtain the best results. The test phase output was also used to develop techniques and skill in interpreting the data configurations obtained from the scaling program. The second phase of this study incorporated the original test data plus data obtained from a larger sample of economists. The hypotheses derived in the first phase were tested against the new data to obtain an indication of their validity. The results of the scaling program were discussed with all respondents who had indicated an interest in seeing the output and the interpretations of the various dimensions were verbally checked against the respondent's interpretations.

The program used for scaling the data was KYST, a program representing a merger of M-D-SCAL 5M and TORSCA 9.

Unless otherwise indicated in this study, the following program control parameters were in effect:

UPPERHALFMATRIX	SFORM=1
DIAGONAL=ABSENT	PRIMARY
NPART=17	REGRESSION=ASCENDING
NREPL1=1	ITERATIONS=50
NREPL3=1	SFGRMN=0.0
TORSCA	SRATST=.999
PRE-ITERATIONS=1	STRMIN=.01
DIMMAX=3	COSAVW=.66
DIMMIN=1	ACSAVW=.66
DIMDIF=1	

The participating respondents in the test phase were members of the Operations Research/Administrative Sciences Department at the Naval Postgraduate School. The second phase included additional members of this department; in addition, the economists at the Defense Resources Management Center at the Naval Postgraduate School were invited to participate in the second phase.

B. DATA COLLECTION

Data used in both phases of this study were collected by means of a pairwise comparison questionnaire (see Appendix A) which requested the respondent to circle the number between 1 and 9 which best represented how similar each given pair was in its general interest to the respondent. The seventeen economics areas used in the study were based

on the classification system for books used by the American Economic Association's Journal of Economic Literature with minor changes as recommended by economists at the Naval Postgraduate School. These areas are listed below, together with the letter which identifies each on the computer output configuration:

- A. Microeconomics
- B. Macroeconomics
- C. History of Economic Thought
- D. Economic Growth and Development
- E. Mathematical Economics
- F. Econometrics
- G. Money
- H. International Economics
- I. Business Economics
- J. Industrial Organization
- K. Agricultural Economics
- L. Manpower and Labor Economics
- M. Welfare Economics
- N. Economic History
- O. Comparative Economic Systems
- P. Regional Economics
- Q. Radical Political Economics

These seventeen stimuli lead to $(17)(16)/2=136$ different pairwise comparisons, each of which represents a cell in the respondent's half-matrix. The pairwise comparisons were not printed on the questionnaire in row or column order, but

randomly distributed within the questionnaire. An additional 16 duplicate comparisons were included to obtain an indication of how consistently the respondent's frame of reference was maintained throughout the questionnaire.

C. ANALYSIS OF RESULTS-TEST PHASE

In the test phase of this study, eight questionnaires (see Appendix A) were distributed, of which seven were completed and returned. Analysis of the 16 duplicated questions indicated relatively consistent responses and revealed no evidence of random or capricious rankings.

To obtain an indication of how well the multidimensional scaling program was providing valid configurations of the respondents' expressions of similarity between the seventeen different areas of economics, the data obtained from this initial group were used to investigate the effects of each of the following operations on the final configurations provided by the scaling program:

- (1) standardizing the input data;
- (2) different treatment of ties in the scaling program;
- (3) different techniques for aggregating data; and
- (4) different starting configurations in the scaling program.

1. Standardizing Input Data

The raw data provided by respondents number 0001 through 0007 was first scaled directly in three, two, and

one dimensions. The raw data were then converted to standard scores and a constant was added to eliminate negative distances through the computer program STANTRIX (see Appendix B). The matrix of adjusted standardized scores for each respondent was then scaled.

The first six columns of Table 1 summarize the results of scaling the raw and standardized data in terms of the final stress obtained in each dimension. In only two cases was there any difference between the final stress in any of the dimensions: the two-dimension solution for respondent 0002 and the one-dimension solution for respondent 0006. Since suboptimal one-dimensional solutions are not unusual in scaling programs, this difference is of little interest. In all other configurations obtained in all dimensions, the results for both raw and standardized scores were practically identical. The small difference (less than three percent) in the two-dimensional stress for respondent 0002 is accounted for primarily by a difference of three points near the center of the configuration (see Figures 16 and 17). All other points were essentially in agreement. Thus, this limited evidence indicates that in scaling similarity percentions obtained through $n(n-1)/2$ pairwise comparisons, it is unlikely that the final configuration will be significantly affected regardless of whether the input data consist of the original ordinal measures or standard scores derived from these raw data.

Respondent Number	Unconstrained Solutions						Constrained Solution		
	Raw Data			Standardized Data			Standardized Data		
	Final Stress			Final Stress			Final Stress		
	Dim. 3	Dim. 2	Dim. 1	Dim. 3	Dim. 2	Dim. 1	Dim. 3	Dim. 2	Dim. 1
0001	.075	.105	.145	.075	.105	.145	.105	.143	.208
0002	.082	.126	.200	.082	.129	.200	.159	.230	.317
0003	.069	.106	.237	.069	.106	.237	.151	.201	.352
0004	.112	.204	.369	.112	.204	.369	.157	.259	.444
0005	.086	.133	.194	.086	.133	.194	.148	.200	.276
0006	.089	.144	.231	.089	.144	.221	.149	.220	.326
0007	.119	.170	.295	.119	.170	.295	.171	.240	.366

Final Stress by Respondent for Various 3-, 2-, and 1-Dimensional Scalings

Table 1

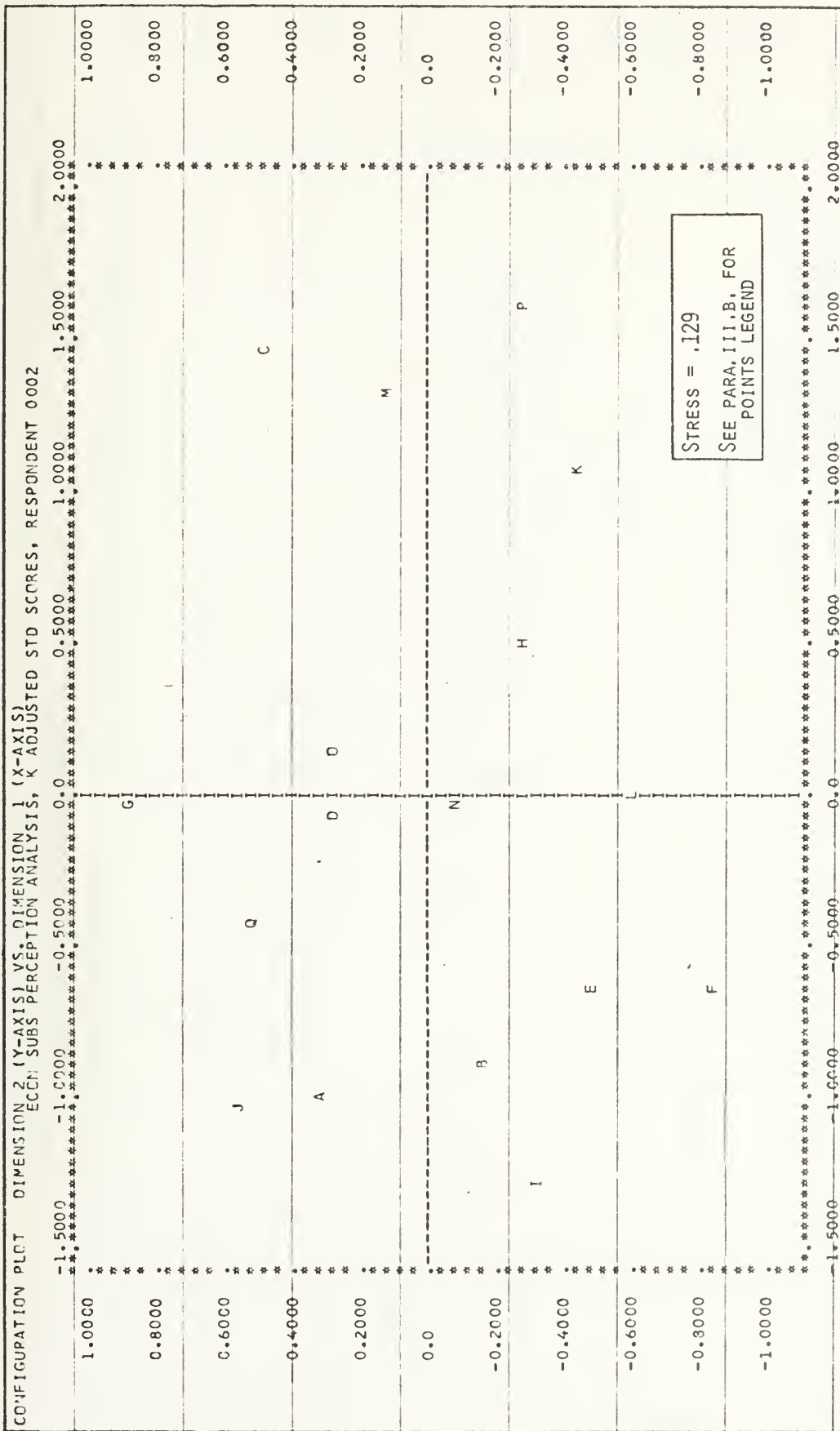


Figure 16

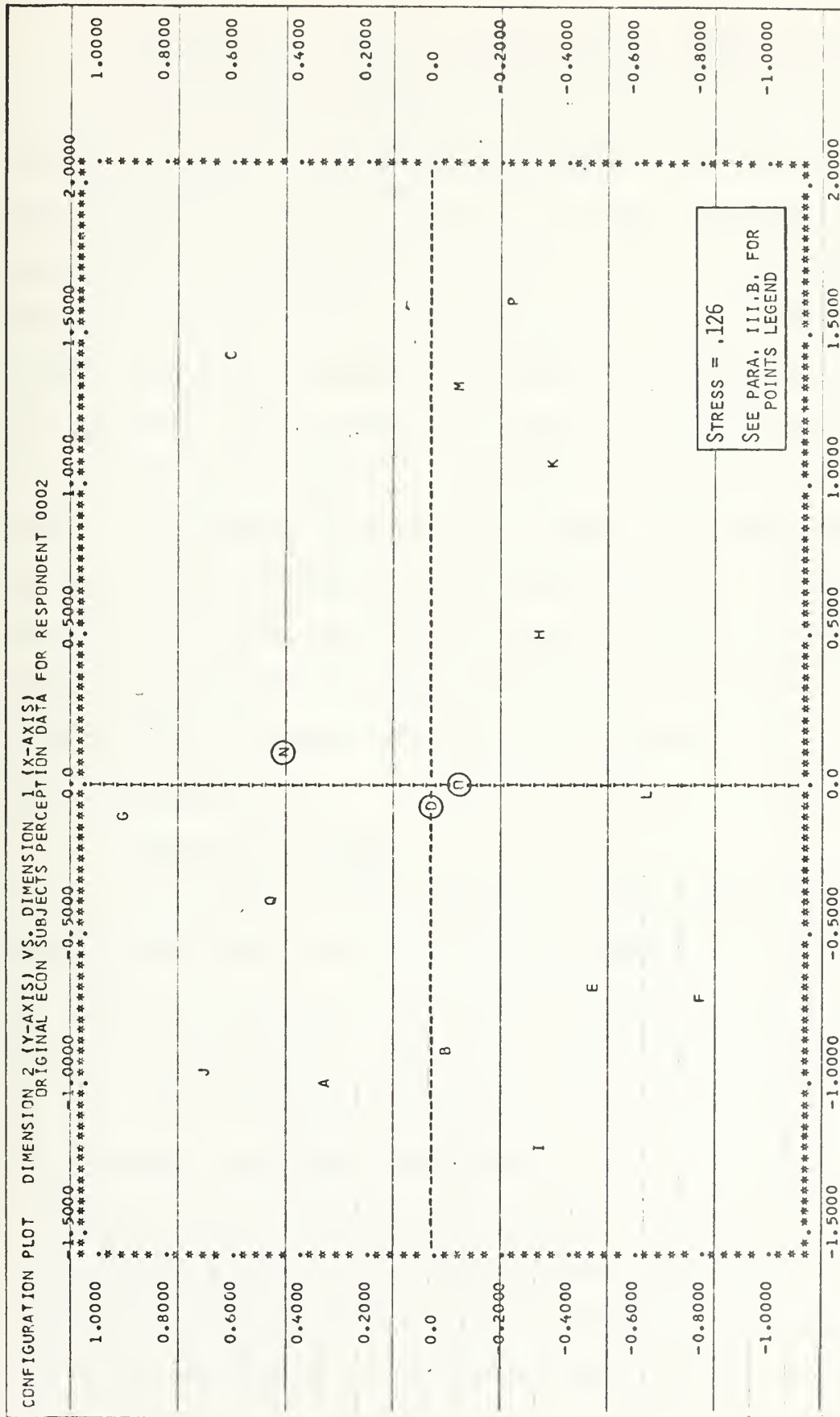


Figure 17

In addition, these results indicate the significance of the stress values obtained in performing comparative scalings. As can be observed from Figures 16 and 17, a numerically very small difference in final stress may be indicative of significant differences in the final configuration. This observations seems to follow from the findings of Spence and Ogilvie [1973], that final stress is a function of the number of points scaled as well as the number of dimensions in which the scaling was performed. In general, the fewer the number of points and the greater the number of dimensions, the lower will be final stress. Thus when scaling only a relatively small number of points (17), small differences in stress can be expected to indicate significant differences in the final configuration.

2. Treatment of Ties

In a questionnaire such as the one used in this study, where there are 136 different judgments to be made and only nine different responses possible on each question, a large number of ties will occur in each respondent's matrix. The KYST multidimensional scaling program provides two different techniques for handling ties. As explained by Kruskal [1964a], the primary approach says that the existence of ties in the original input data (the s_{ij} 's) should not, in itself, force the configuration's interpoint distances (the d_{ij} 's) to be equal. The scaling algorithm

accomplishes this by not constraining the values of \hat{d}_{ij} . Thus the primary approach can be described as follows:

if $s_{ij} = s_{kl}$ then $\hat{d}_{ij} < d_{kl}$, or $\hat{d}_{ij} = \hat{d}_{kl}$, or $\hat{d}_{ij} > \hat{d}_{kl}$.

The secondary approach to ties presumes that the existence of any condition $s_{ij} = s_{kl}$ is evidence that the distance d_{ij} should equal d_{kl} . This constraint is accomplished by requiring, for each $s_{ij} = s_{kl}$, that $\hat{d}_{ij} = \hat{d}_{kl}$.

Kruskal's explanation suggests that if our input data are metric, the equality constraint (the secondary approach) should result in a more reliable configuration, whereas if the input is nonmetric and represents only coarse category judgments, the unconstrained approach (the primary approach) should lead to better results.

To empirically test this hypothesis, a circular configuration of sixteen points, equally spaced about the perimeter, and all equidistant from a seventeenth center point, was drawn (Figure 18). Each of the 136 interpoint distances was computed and the resulting distance matrix was used as input into the scaling program. Using the "primary" approach to ties, the program terminated when minimum stress of .019 was achieved. The resulting configuration (Figure 19) closely resembled the original configuration; however, recovery was not exact. Because the program's Y-axis compression results in a misleading elliptical representation, the "roundness" must be checked

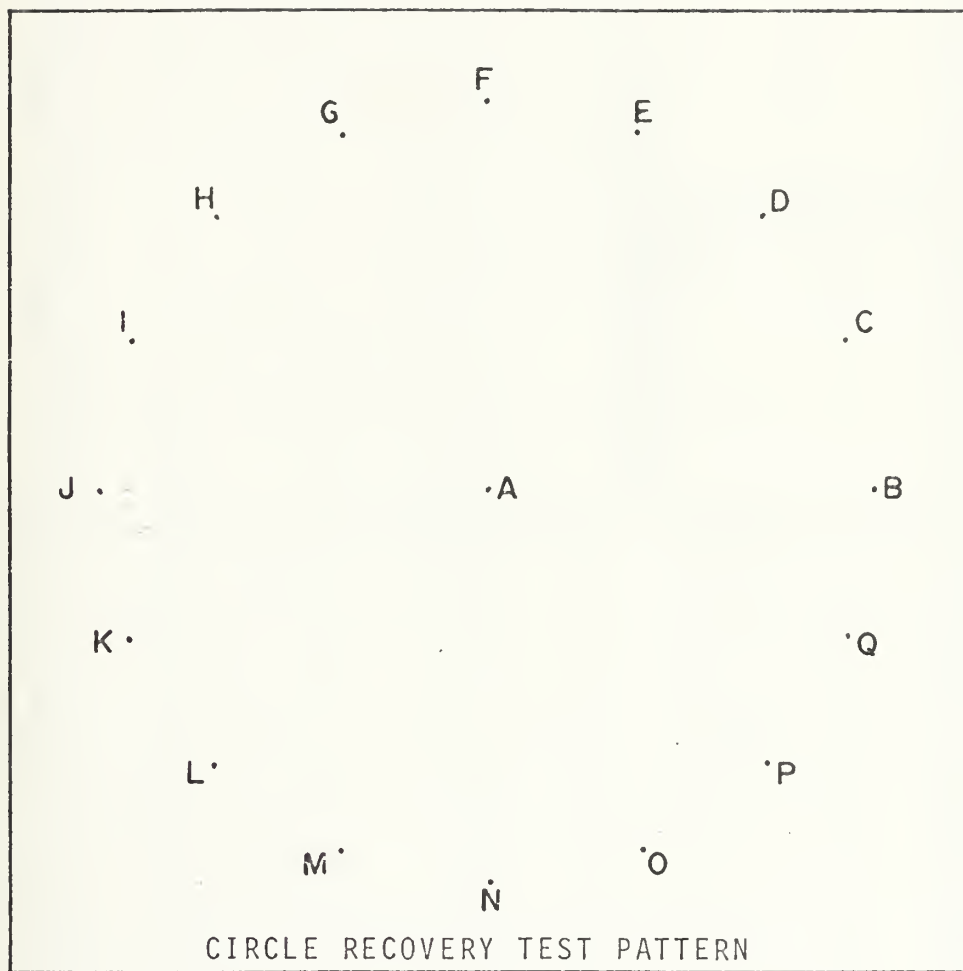


Figure 18

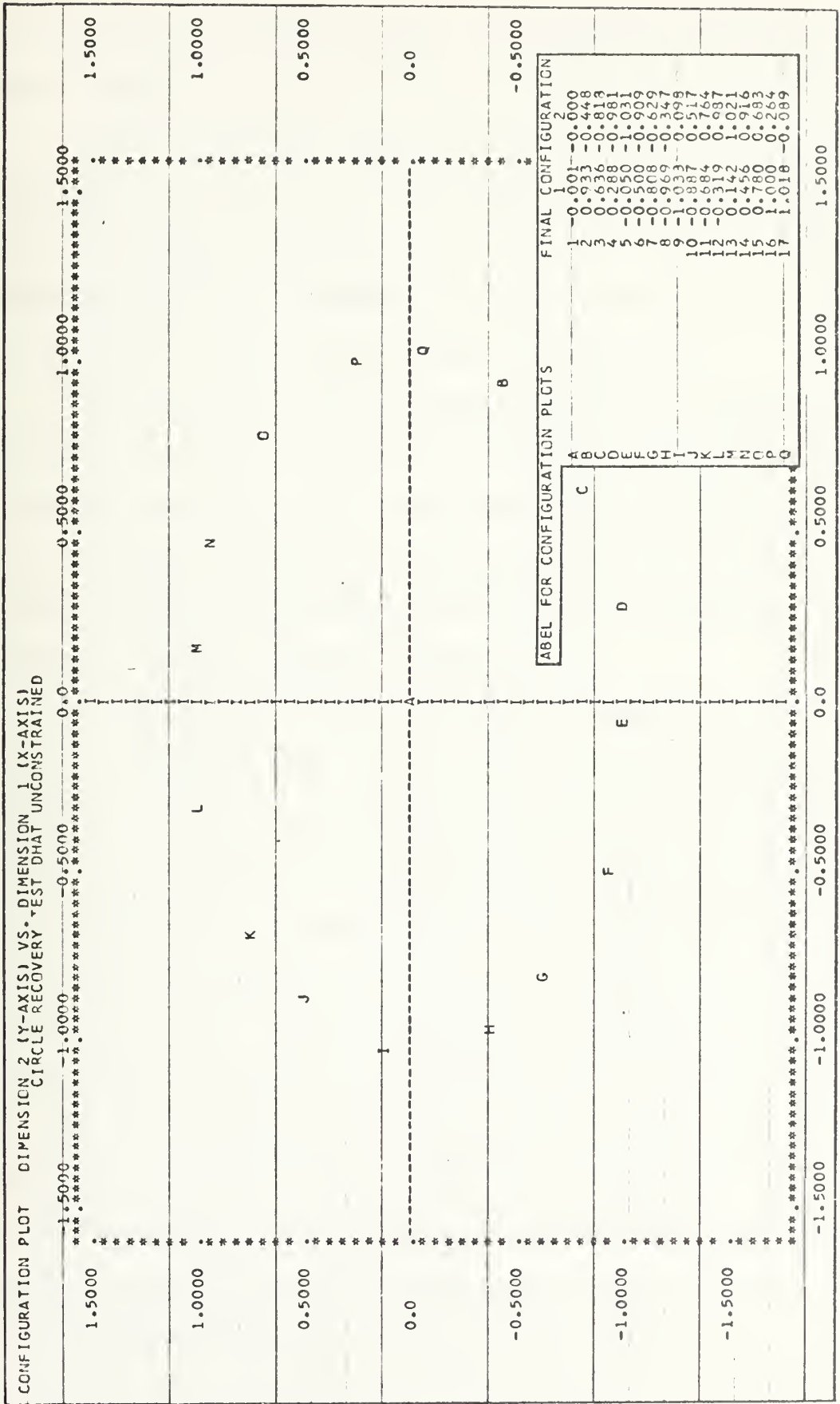


Figure 19

by referring to the coordinates of each point reproduced on the figure.

The "secondary" approach also terminated when minimum stress of .019 in two-dimensions was achieved. However, the resulting configuration, shown in Figure 20, was an essentially perfect recovery of the original figure, as can be verified by checking the points coordinates.

The adjusted standardized dissimilarity measures of the first seven respondents were then scaled using the secondary approach. The results are shown in the last three columns of Table 1. In every instance the final stress obtained was greater than the stress obtained using the primary approach (columns 1-6).

This evidence suggests that the use of the secondary approach with psychological distance input data is likely to cause some degradation of the resulting configuration, especially when there are a large number of ties. Obviously, if there are few ties, the results are unlikely to be significantly affected regardless of which approach is used. Since most applied researchers will be dealing with perceived (or psychological) distances rather than metric distances, Kruskal's [1964a] opinion that the primary approach seems preferable is validated by these tests. Based on these results, it appears that the primary approach to handling ties is preferable unless the original data consist of fairly precise metric distances.

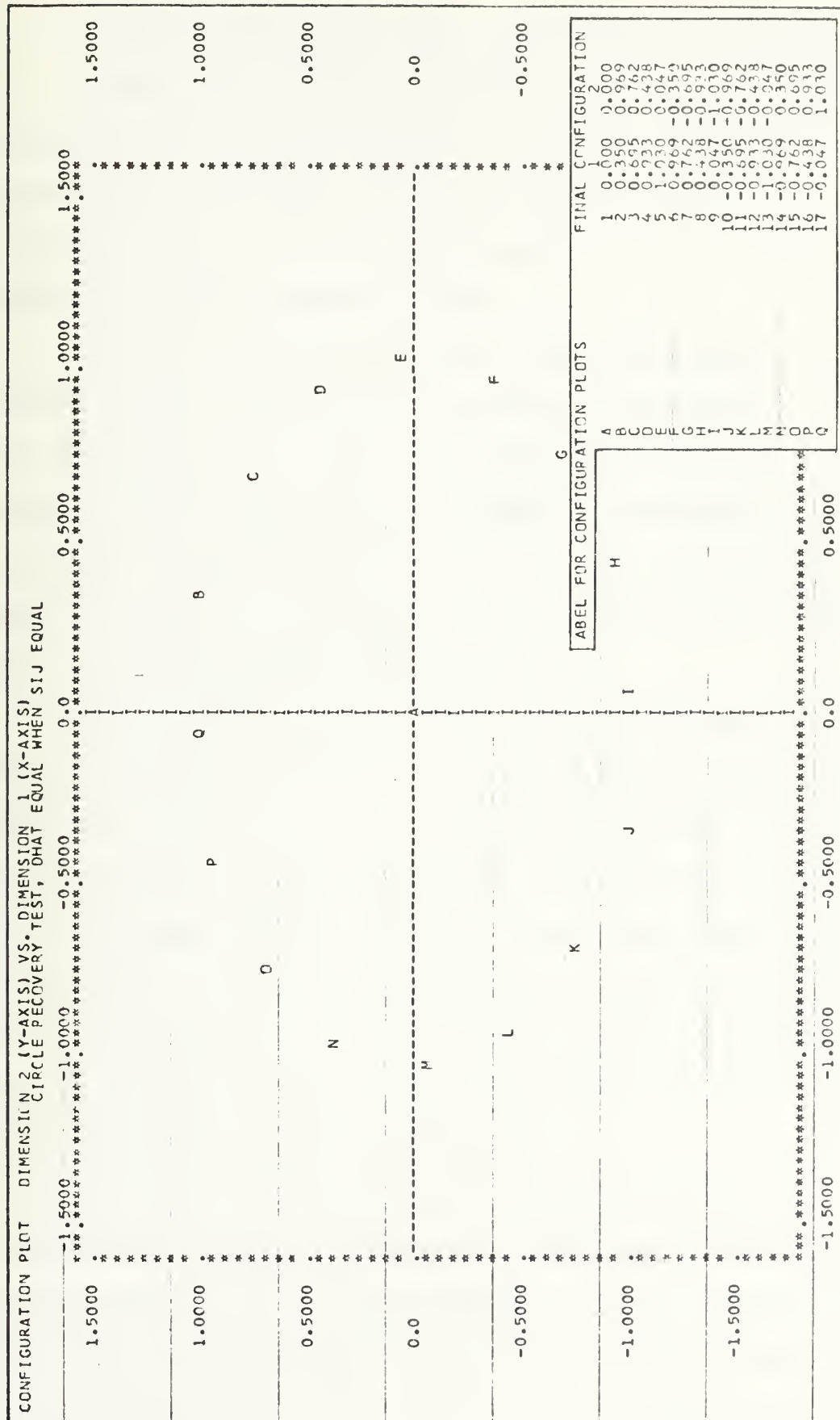


Figure 20

3. Methods of Aggregation

Data collected for the first seven respondents were aggregated using four different techniques. Three of the aggregations were performed by the program STANTRIX prior to scaling data. These are described in detail in Appendix B and are briefly summarized here.

The first aggregate matrix (labeled AGNEW in the program and in following discussions) is produced by summing the standard score in each cell across all individual matrices, then dividing by the number of individual matrices to obtain the mean standard score for each cell in the aggregate matrix. A constant is then added to each matrix cell to eliminate negative distances.

The second aggregate matrix (labeled AGRAW) is produced by summing the raw score in each cell across all individual matrices, then dividing by the number of individual matrices to obtain the mean raw score for each cell in the aggregate matrix. Standard scores are then computed for each cell in the aggregate matrix and the average raw score for each cell is then replaced by the corresponding standard score to which the aggregate matrix mean has been added to eliminate negative distances.

The third aggregate matrix (labeled ADSTD) is produced by summing the standard score, after the constant has been added (the adjusted standard score), in each cell across all individual matrices, then dividing by the number

of individual matrices to obtain the mean adjusted standard score for each cell in the aggregate matrix. Standard scores are then computed for each cell in the aggregate matrix and the mean adjusted standard score in each cell is replaced by the standard score to which the aggregate matrix mean has been added to eliminate negative distances.

The fourth type of aggregation is performed by using the SPLIT=NOMORE option in the KYST scaling program and using all individual respondents' data decks as input. The scaling program treats each respondent's perceptions as replicated observations of the dissimilarity between stimuli i and j and seeks a configuration which minimizes final stress over all respondents' observations. Two types of input data were used for this aggregation, the standard scores of each respondent, to which a constant had been added; and the originally provided raw scores of each respondent.

This part of the study was an investigation into the apparent validity of different methods of aggregating data as measured by final stress achieved in each dimension. Table 2 portrays the results for the first seven respondents using each of the four types of aggregation.

Based on final stress in each dimension, the method of aggregation which sums each cell across all respondents' original response matrices, averages these scores, then standardizes and adds a constant to each aggregate matrix

cell (the AGRW matrix) might appear to produce the best overall representation of the respondent's perceptions. The KYST aggregations also follow the pattern of producing lower stress with raw data than with standardized data. However, in recalling the way in which the input data were derived, we can legitimately question whether this lowest stress solution portrays the best overall configuration.

<u>Aggregation Method</u>	<u>Final Stress</u>		
	<u>Dim. 3</u>	<u>Dim. 2</u>	<u>Dim. 1</u>
AGNEW	.131	.215	.354
AGRAW	.126	.207	.323
ADSTD	.131	.215	.355
KYST, using std. scores	.294	.343	.505
KYST, using raw scores	.222	.313	.460

Final Stress Achieved with Five Different
Types of Aggregation

Table 2

To better understand this question, consider two different respondents, A and B, who both share quite similar perceptions of economic subjects but with quite different personalities. Respondent A might respond to most of the pairwise comparisons with answers in the range of "3" to "7," with only a rare "2" or "8." Respondent B's responses, on the other hand, might range freely over the scale from

"1" to "9." By standardizing the scores of each respondent, we are in effect putting both sets of scores on the same standard scale, thus allowing a reliable "average" aggregate matrix to be computed. Simply averaging the raw scores of respondents A and B before adjusting their scores to the same scale might lead to questionable results.

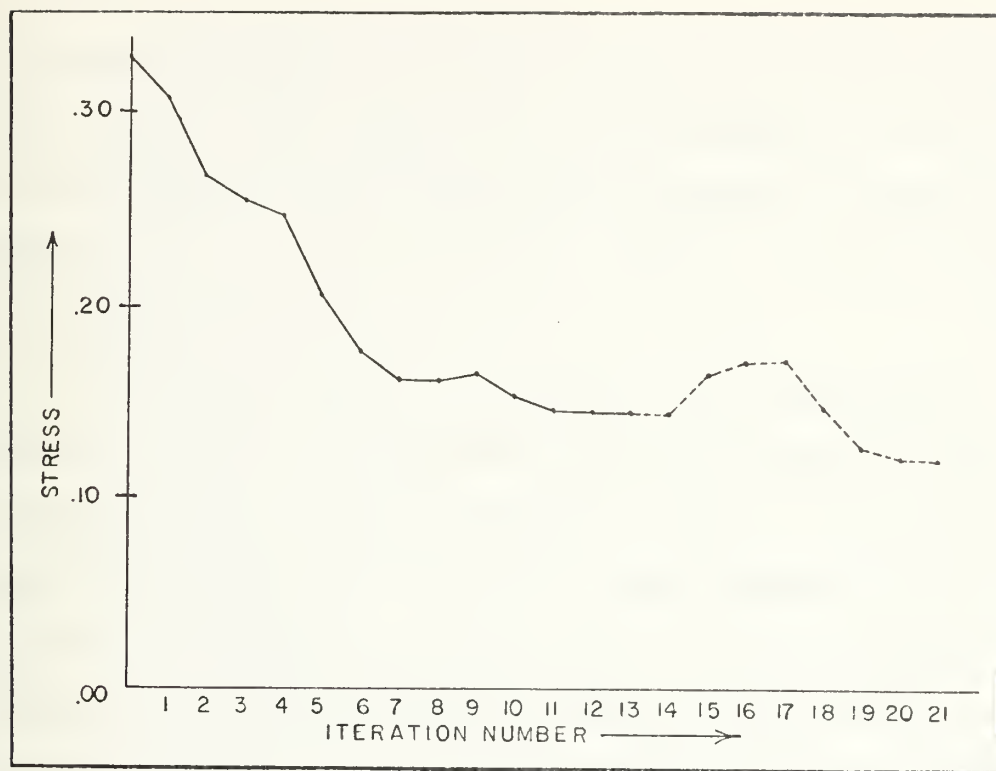
In the question at hand, that of determining which is the best aggregate configuration, the answer seems clearly only the solutions based on standardized scores can be statistically justified. Therefore, we must conclude that the two configurations based on standardized scores in which the data are aggregated prior to scaling (the AGNEW and ADSTD matrices) represent the best overall configuration shared by all respondents in the "test" phase of this study, despite the fact that the solutions obtained using raw data were characterized by lower stress.

On the surface, this conclusion might seem to contradict what has been suggested earlier, that the lower the final stress, the better the configuration. However, it must be kept in mind that stress is only a measure of the "badness of fit" of the input data. If the input data do not accurately reflect the underlying configuration, no scaling of the data is likely to produce accurate results regardless of how low is the final stress value obtained. Thus our apparently "better" results obtained by scaling aggregated raw data can be viewed as simply another manifestation of the well-known adage among computer programmers,

"garbage in, garbage out." More importantly, this case is a good example of the importance, stressed by many students of multidimensional scaling, of not being blinded by stress as the only measure of configuration recovery, but to clearly understand the validity of the input data as well as the operation of the scaling program under different input parameters. Sherman [1972], Klahr [1969], and Spence and Ogilvie [1972] have all demonstrated that impressively low values of stress may be obtained in a scaling program with a small number of random points in a few dimensions. Their findings, and this interpretation of the aggregate results do not damage the theory behind multidimensional scaling. They do remind the user that the program does not correct for lack of good judgment on the part of the user.

4. Different Starting Configurations

As described earlier, the multidimensional scaling algorithm seeks to obtain the best configuration of the original dissimilarity data by iteratively moving the points in the configuration to minimize the value of stress while maintaining a monotonic relationship among the d_{ij} 's. There is always a possibility that this process, in which the points are moved by small amounts on each iteration, can lead to an invalid configuration.



Stress by Iteration Number, Hypothetical

Figure 21

To illustrate, Figure 21 is a hypothetical plot of stress on each iteration of the scaling program. Moving the points around on the seventh and eighth iterations did not improve the stress, and the ninth actually increased it. However, the changes made on the tenth iteration resulted in a significant decrease in stress which remained constant during the next few iterations. The program, "sensing" that a configuration had been reached from which no further improvement was possible, might likely terminate after the fourteenth iteration. However, it is possible that had the program continued, some additional movement of points might result in a large increase

in stress followed by a sharp decrease to an even lower stress value. By stopping on the fourteenth iteration in this example, the program would produce a suboptimal configuration, having mistakenly sensed a local minimum as the global minimum.

Although Figure 21 appears to be a plausible scenario, Shepard [1972] states that this has not been a serious problem except in special cases such as one-dimensional solutions and "city block" metrics ($\rho = 1$). However, as a check on the validity of the configurations, a number of different random starting configurations were specified and the final results compared with the results obtained from using the standard initial configuration which uses a technique (TORSCA) to "pre-scale" the data and obtain a low-stress starting configuration.

This test was conducted in two parts. In the first part one of the suboptimal solutions of aggregated input data for the initial group of respondents (the ADSTD matrix) was used as the basis for comparison. The second part used the poorest individual results based on final stress values (those for respondent 0004) as the benchmark. The scaling program provides the option of establishing random starting configurations for the data. A method of selecting a large number of different random initial configurations is supplied within the program; the user specifies the number which are to be discarded before the group of points

required to represent the configuration is chosen. By changing this number of "discards," the user can obtain new and different starting configurations on successive runs.

For the aggregate data test, fifteen different random starting configurations were specified. Input control options were standard on all tests for local minimum with the following exceptions:

RANDOM=(INTEGER-different on each scaling run)

ITERATIONS=100

None of the computer scaling runs required as many as 100 iterations. All terminated when an apparent minimum stress was achieved. The results of the aggregate test runs are shown in Table 3. The random integer used to specify the starting configuration, the initial and final stress in each dimensional solution, and the number of iterations required to reach the minimum stress solution are shown. The results obtained from using the standard TORSCA initial configuration are also shown for comparison. In one scaling (using RANDOM=18) an invalid configuration was obtained in the three-dimension solution. Apparently in this scaling run, the program mistook the three-dimensional local minimum for a global minimum; however, reasonably good local minimum were obtained in the two- and one-dimensional solutions on the same run.

Two observations can be generalized from Table 3. First, with random starting configurations, the final stress

Starting Configuration	Dimension 3			Dimension 2			Dimension 1		
	Initial Stress	Final Stress	Itera- tions	Initial Stress	Final Stress	Itera- tions	Initial Stress	Final Stress	Itera- tions
26	.435	.131	97	.242	.215	15	.368	.355	14
18	.470	.248	21	.362	.228	58	.370	.354	14
2	.429	.161	49	.237	.217	15	.377	.358	15
1	.480	.140	33	.248	.225	15	.370	.354	14
7	.500	.144	83	.251	.217	17	.376	.356	14
15	.458	.144	42	.262	.228	16	.372	.354	14
58	.415	.140	66	.245	.219	16	.374	.357	14
56	.446	.142	42	.245	.215	16	.368	.355	14
67	.391	.134	62	.251	.228	15	.372	.354	14
94	.342	.148	39	.253	.215	18	.369	.355	14
91	.340	.162	38	.229	.218	15	.368	.356	14
41	.458	.169	40	.238	.223	16	.370	.355	15
8	.433	.133	39	.250	.215	15	.368	.355	14
9	.460	.131	47	.243	.215	15	.368	.355	14
99	.421	.133	36	.266	.215	17	.368	.355	14
"TORSCA" starting configuration	.194	.131	28	.244	.215	15	.368	.355	14

Results of Random Initial Configuration Scalings of
Standardized Aggregated Scores

Table 3

of the configuration apparently is not a function of the stress of the initial configuration. For example, initial stress of .415 with RANDOM=58 led to a final stress (in three dimensions) of .140, while initial stress of .460 with RANDOM=9 led to final stress (in three dimensions) of .131. Second, only in the one-dimensional solution was a randomly generated starting configuration able to improve upon the results obtained with the standard TORSCA initial configuration, and this was only on the order of .001. For the three- and two-dimensional solutions, the final stress values using random starting configurations were generally higher than those obtained with the standard TORSCA starting configuration.

Scaling the data for respondent 0004 repeatedly, using different random starting configurations produced similar results, as shown in Table 4. No random start scaling produced better results in three dimensions than did the standard TORSCA starting configuration and only one of the two-dimensional solutions produced lower stress (by .002). Eight of the one-dimensional solutions produced lower final stress, of from .006 to .002, than did the TORSCA initial configuration.

In addition, the observations made concerning the lack of any relationship between initial and final stress using random starting configurations appear justified. An initial stress of .343 (RANDOM=399), led to final stress

Starting Configuration	Dimension 3			Dimension 2			Dimension 1		
	Initial Stress	Final Stress	Itera- tions	Initial Stress	Final Stress	Itera- tions	Initial Stress	Final Stress	Itera- tions
18	.438	.130	45	.230	.204	25	.383	.371	14
2	.412	.121	40	.245	.204	25	.377	.366	15
15	.457	.112	51	.230	.204	16	.376	.369	15
58	.407	.118	46	.233	.208	15	.383	.369	14
1	.477	.120	44	.236	.204	25	.377	.366	14
4	.467	.112	48	.232	.204	15	.378	.369	14
9	.447	.119	59	.233	.202	16	.388	.371	14
99	.403	.136	35	.230	.210	16	.378	.367	14
7	.485	.127	48	.239	.204	24	.386	.371	14
73	.363	.116	40	.247	.217	18	.386	.363	15
399	.343	.137	70	.234	.207	18	.378	.366	14
39	.462	.112	68	.232	.204	16	.376	.369	15
25	.412	.128	47	.236	.217	21	.377	.365	14
57	.462	.116	58	.236	.217	17	.385	.363	15
36	.442	.135	29	.229	.204	19	.378	.365	16
"TORSKA" starting configuration	.161	.112	19	.232	.204	15	.378	.369	14

Results of Random Initial Configuration Scalings of Standardized
Scores, Respondent 0004

Table 4

in three dimensions of .137; while initial stress of .457 (RANDOM=15) led to final stress of .112.

Based on these studies, it appears that unless the user is interested in the one-dimensional solutions, the standard TORSCA initial configuration will likely lead to the best results. The comment of Kruskal, Young, and Seery in the KYST monograph (undated) that use of the TORSCA initial configuration reduces the chance of reaching a local rather than the global minimum seems well justified by this study.

IV. INTERPRETATION OF INDIVIDUAL AND AGGREGATE RESULTS

Following the study of the scaling program's operation and the legitimacy of output obtained under different program control parameters and types of input, the questionnaire in Appendix A was distributed to the remaining seven economists in the Operations Research/Administrative Sciences (OR/AS) Department and to nine economists at the Defense Resource Management Center (DREMC) at the Naval Postgraduate School. All of the OR/AS economists returned signed questionnaires, and eight of the nine distributed to DREMC economists were completed and returned, of which six were signed. Thus, including the eight questionnaires utilized during the test phase, there were twenty-two questionnaires completed and returned out of twenty-four distributed for a response rate of 92%. Two of the questionnaires were returned unsigned, indicating that the respondents did not wish to see the results from the scaling program, and one of the respondents was not available for further participation in the study.

The responses on the sixteen "check questions" in the questionnaires returned by the respondents in the second phase of this study were then compared with the responses on their counterpart questions. One of the unsigned questionnaires showed evidence of capricious responses (about 21% of the pairs were ranked "5" and the remainder "9") and was therefore eliminated from the study. There was no

evidence of grossly inconsistent or random or capricious responses among any of the other questionnaires. Thus there was a total of twenty economists whose questionnaires were scaled and analyzed, thirteen in the OR/AS Department and seven at DREMC.

After scaling all individual respondents' questionnaires, three different aggregations were performed. All OR/AS economists' responses were aggregated; all DREMC economists' responses were aggregated; and an aggregation of all respondents in both the OR/AS Department and DREMC was performed. OR/AS and DREMC responses were aggregated separately because it was considered possible that the teaching orientation and type of student taught by each group might influence the respondents' perceptions of economic subjects. The OR/AS Department is primarily involved in teaching middle management level military officers (lieutenants junior grade to commanders) in an accredited, formally structured program leading to a master's degree in management. The DREMC is primarily oriented toward conducting short courses (two weeks or so) for high level military and civilian managers in DOD and from foreign nations. While both the OR/AS and the DREMC courses emphasize the applicability of economics to the military policy and decision making processes, the OR/AS courses tend to contain a high proportion of economic theory to buttress the applied aspects of the subjects, while the DREMC courses tend to focus primarily on the applied aspects of economics which are most relevant to the needs of the students.

During the test phase of this study it was noted that both raw scores and standardized scores led to essentially identical three-dimensional configurations. The results of scaling the additional data collected in the second phase of this study, shown in Table 5, confirmed this observation. The final stress in all dimensions is practically identical, and there were no meaningful differences between a respondent's three-dimensional configuration using standardized data input and the same configuration using raw data input.

A. INTERPRETATIONS OF INDIVIDUAL CONFIGURATIONS

Recalling the discussion in Section II, the appropriate number of dimensions for scaling can be determined in several ways. In general, if there is a clearly defined elbow in the curve of final stress versus number of dimensions, the dimension at which the elbow occurs is the appropriate number of dimensions for scaling. In the absence of a clear elbow, the number of dimensions should be set at the maximum number of dimensions which are interpretable, generally not more than three.

Scaling the responses in the test phase of this study had revealed no clearly defined elbow; thus all were scaled in three dimensions with the hope that respondents could identify a meaningful relationship among the points in each dimension. The three-dimensional solutions consisted of three separate configuration plots for each respondent: a two- versus one-dimensional plot, a three- versus

<u>Respondent</u>	<u>Raw Scores</u>			<u>Standard Scores</u>		
	<u>DIM 3</u>	<u>DIM 2</u>	<u>DIM 1</u>	<u>DIM 3</u>	<u>DIM 2</u>	<u>DIM 1</u>
0101	.092	.132	.220	.092	.132	.220
0102	.122	.197	.387	.122	.197	.387
0103	.087	.129	.197	.087	.129	.197
0104	.052	.099	.197	.052	.099	.197
0105	.010	.008	.009	.010	.007	.010
0106	.059	.092	.141	.059	.092	.141
0107	.000	.003	.005	.000	.003	.006
0201	.112	.169	.289	.112	.169	.289
0202	.047	.083	.128	.047	.083	.128
0203	.095	.145	.270	.095	.145	.270
0204	.105	.188	.349	.105	.188	.349
0205	.008	.024	.008	.008	.025	.009
0206	.058	.115	.187	.058	.115	.187
0207	.103	.173	.307	.103	.173	.307
0208	.101	.173	.334	.101	.173	.334

Final Stress by Dimension -
Raw Scores and Standard Scores

Table 5

one-dimension plot, and a three- versus two-dimension plot.

Based on the assumption that the individual respondent would be best qualified to interpret his own configuration, appointments were made to discuss the results with each respondent individually, in the same order listed above. The respondent was first asked if he could assign a name to the first dimension, by studying the relationships among the points in the horizontal plane of the plot. If he had difficulty in so doing, he was asked to imagine collapsing the points onto the x-axis and try to describe what made the end points different. If this did not help, he was asked to consider the various groups of points and try to determine what made the groups different from one another and the points within each group similar. Using the same plot and the same procedure he was then asked to attempt to identify the second dimension. He was then shown the three- versus one-dimension plot and asked to attempt to identify the third dimension. Finally, he was shown the three- versus two-dimension plot. If he had identified dimensions two and three on previous plots, he was asked to use this configuration to verify the labels he had assigned to dimensions two and three. If he had not been able to identify dimension two or three, he was asked to try again, looking at the points from this different perspective. (No respondent in the test phase was unable to identify the first dimension.)

The results of discussing these configuration plots with the thirteen OR/AS economists and the six DREMC economists who had signed their questionnaires are reported in Table 6. (One of the OR/AS respondents was not available for an interview. His responses were included in the total aggregated scalings, however.) The majority of the respondents (13 of 19) indicated that the descriptor "interest" or "preference" best accounted for the orientation of the points in the first dimension. While six of the respondents were unable to identify the second dimension as meaningful, and nine were unable to identify the third dimension, no respondent who had identified the first dimension as something other than interest or preference was unable to fully identify dimensions two and three. This may indicate the existence of a relationship between the richness and complexity of an economist's perceptions of economic subjects and their logical ordering within his perceptual space. The possible validity of this hypothesis will be examined later in this study.

During the individual interviews of the test phase it became apparent that there were apparently many factors external to the content of the individual economic subjects which influenced the respondents' rankings or interpretations. These are briefly discussed below.

Two respondents found a clear pattern in the second dimension on the "preferred" side of the first dimension. However, they saw no relationship among the points on the

SUMMARY OF ALL INTERVIEWS

<u>Respondent</u>	<u>Dimension 1</u>	<u>Dimension 2</u>	<u>Dimension 3</u>
OR/AS Dept:			
0002	interest	-----	-----
0003	preference	-----	-----
0004	analytical/ descriptive	prestige or popularity or professional status	macro/micro
0005	preference	importance (preferred side only)	positive/normative
0006	interest	macro/micro	"one-answer" models/ "many-answer" models
0007	interest	-----	-----
0101	interest	-----	-----
0102	preference	usefulness	theoretical/applied
0103	preference	knowledgeability in the subject	-----
0104	difficulty of subject matter	purity/political orientation (or contamination)	relevance to problem-solving

Table 6

SUMMARY OF ALL INTERVIEWS

<u>Respondent</u>	<u>Dimension 1</u>	<u>Dimension 2</u>	<u>Dimension 3</u>
OR/AS Dept:			
0105	purity/political orientation (or contamination)	interest	relevance to problem-solving
0106	not available for interview		
0107	interest	-----	-----
<u>DREMC:</u>			
0202	preference	importance (preferred side only)	-----
0203	interest	importance	appropriate for quantification ("important" side only)
0204	usefulness to the profession	theoretical/practical	preference
0206	interest	-----	-----
0207	preference	core subjects/ non-core subjects	-----
0208	empirical	reliance on math. models	macro/micro

Table 6 (continued)

"least-preferred" side. Four other respondents who were unable to interpret the second dimension indicated that they were not very familiar with some of the subjects on the "least-preferred" side. This led to an investigation of the importance of familiarity as a prerequisite for meaningful configurations, a subject discussed in Section V of this study.

Referring back to the chart of final stress by dimension for each respondent (Table 5) three respondents submitted questionnaires which resulted in configurations with remarkably low final stress. It has already been mentioned that respondent 0205 ranked all pairs using only "5" and "9." This binary approach to the questionnaire resulted in the extremely low final stress values. The interviews with the other two respondents with final stress of .01 or less were more interesting.

The interview with respondent 0107 indicated that he apparently misinterpreted the directions and utilized a mechanistic approach to ranking the pairs. While he did not choose to describe his technique, he apparently listed each of the seventeen subjects in order of preference, and then assigned a number beginning with one for the most preferred subject, to each subject (or group of subjects). He then was able to rank each pair presented by the absolute difference of their ordinal preference position. While this led to valid results and perfect (zero) stress in the first

dimension, it precluded any meaningful configuration in dimensions higher than one.

The responses of respondent 0105 provided a very interesting configuration when scaled. Radical Political Economics was positioned on the far end of the x-axis of dimension one and all other subjects were arrayed vertically at the other end within a very narrow band running from top to bottom of the second and third dimensions, when plotted against dimension one. The respondent found it very easy to interpret all three dimensions presented. The strong perceptions of the respondent concerning Radical Political Economics as expressed in his similarity rankings enabled the program to attain an extremely low stress, for the respondent perceived sixteen subjects as economics and one as politics. Thus, his second and third dimension perceptions were the only valid indicators of his perceptions of "economic" subjects. This result is interesting for its illustration of the importance of including in a questionnaire only subjects which can all be perceived as belonging to the same class or group. To do otherwise forces the respondent to change his frame of reference from one group to another as he compares the subjects.

Two respondents stated that their perceptions of one or more of the subjects were strongly influenced by their exposure to those subjects during graduate school. Several respondents indicated or stated that certain subjects ranked unusually high on their "interest" or "preference" scale

because they were teaching (or had recently taught) or were advising theses pertaining to these subjects. These comments indicate the existence of both long-term and short-term influences in personal preferences. In the short run, preferences may change daily or hourly, depending on the interests of the moment. In the long run, the prior impressions of an economist, especially those gathered during graduate school, appear to strongly influence his perceptions. It would seem logical that this influence would decrease the longer an economist was away from graduate school, and it was, in fact, two of the youngest economists who mentioned this influence. In the same regard, it would appear possible that the interests of the older economists might have become more focused on the group of subjects with which they were most interested and their familiarity with fields in which they had found little application or importance would tend to decrease over time. This again leads to the problem of familiarity with the subjects which will be discussed later. It also illustrates the fact that any configuration of points derived through this scaling program is valid only for the point in time at which it was obtained. As one moves further away from that point, the less reliable the results are likely to become. Since the configuration does not differentiate between current interests and deeply-rooted preferences, if one wishes to find a configuration of the long-run perceptions

of economists, it may be necessary to utilize techniques to discount the influence of short-term factors.

B. AGGREGATE RESULTS

Following the interviews with all respondents and analysis of the individual configurations, the individual responses were aggregated through the program STANTRIX into three groups, all respondents in the OR/AS Department, all respondents at DREMC, and the aggregate responses from all respondents at the Naval Postgraduate School, both in the OR/AS Department and at DREMC. The aggregated standardized scores obtained from the matrix ADSTD were then scaled in three dimensions, using standard KYST program control options, and an attempt was made to interpret each dimension.

The results of aggregating the respondents in the OR/AS Department are shown in Figures 22 and 23. Figure 22 is dimension two versus dimension one. Looking at the first dimension, a highly subjective interpretation is that this is a quantitative, non-quantitative dimension. Although two points, Mathematical Economics (E) and Agricultural Economics (K), seem to be misplaced if the "quantitative" descriptor is accepted, no other name seems to better describe the first dimension.

Still looking at Figure 22, but focusing our interest on the location of the subjects within the second dimension, we find Welfare Economics (M) at one end of the scale and Monetary Economics (G) at the other. This dimension can

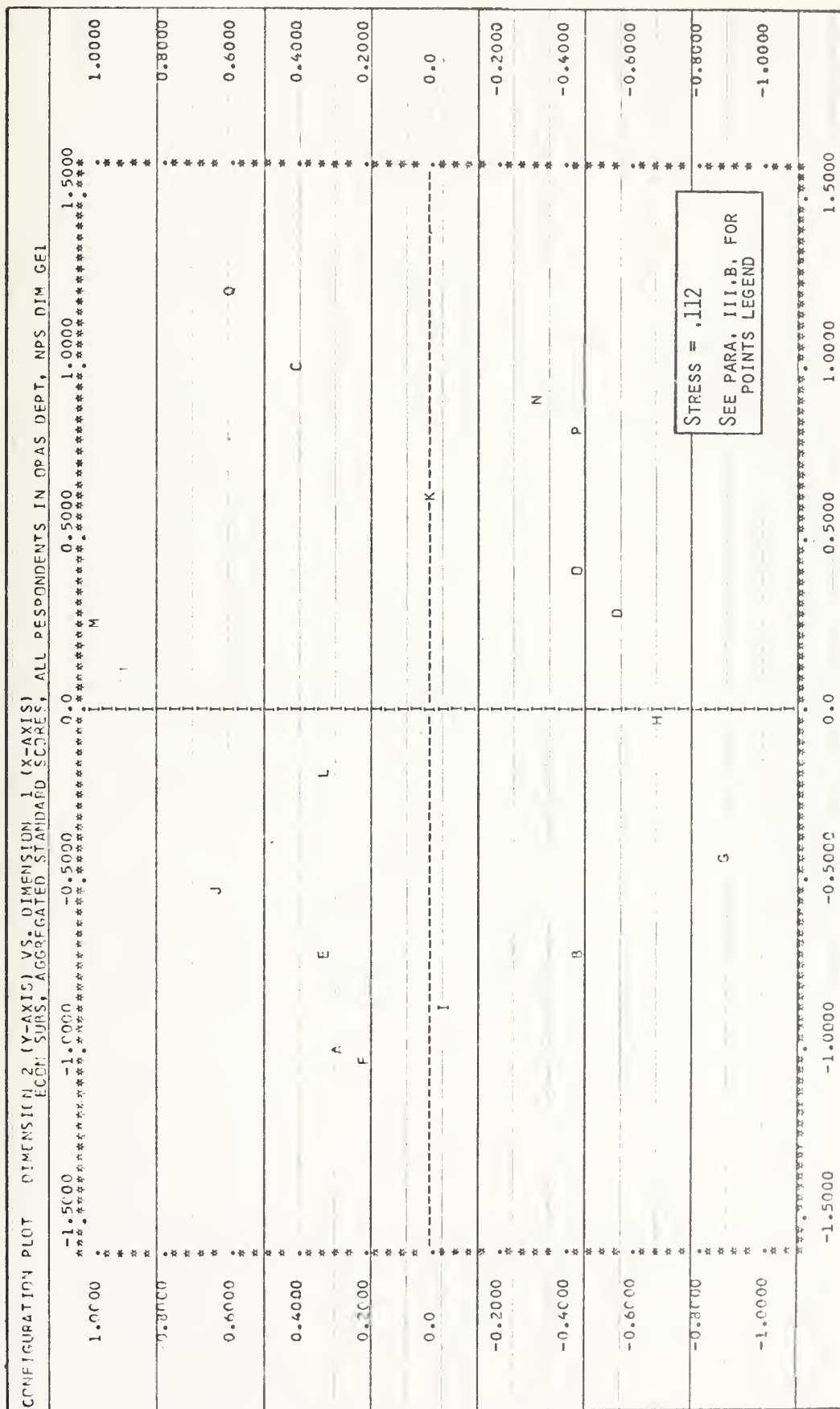


Figure 22



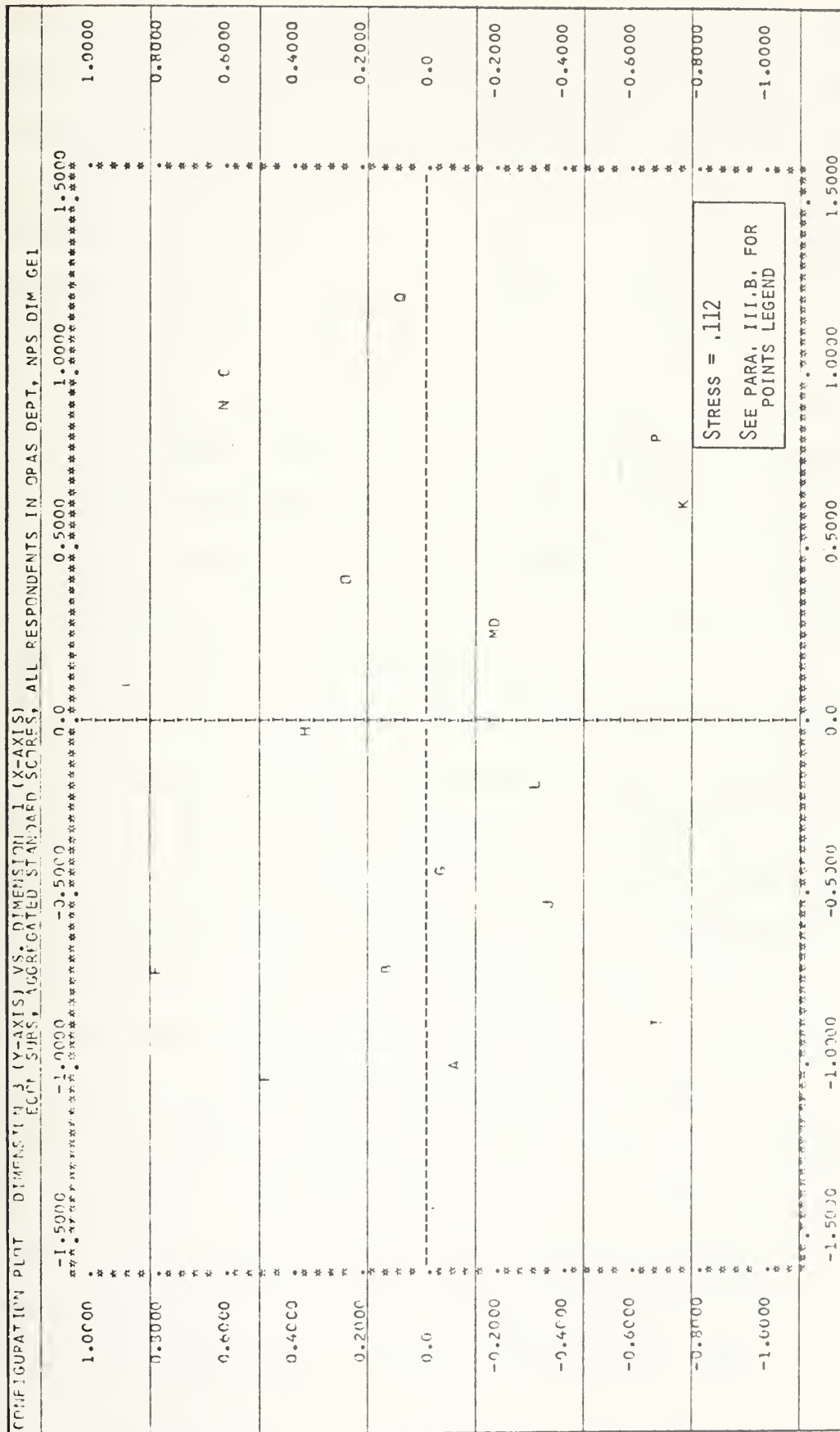


Figure 23

probably best be described as "positive/normative." The subjects toward the top of the scale seem to have in common a concern with amelioration of the human condition; an interest in what "could be." The subjects toward the bottom generally are focused more on "what is;" a more pragmatic outlook. The large difference between History of Thought (C) and Economic History (N) also seems to bear out this description. History of Economic Thought emphasizes the contributions of original thinkers to the improvement of economic theory, while Economic History describes "what was" in terms of its economic effects. Depending upon one's perceptions of the subjects, this "positive/normative" descriptor seems to fit the second dimension well.

Looking now at Figure 23, the third dimension appears to be a "theoretical/applied" dimension. If we consider first the eight leftmost points, Mathematical Economics (E) is highest, followed by Econometrics (F), Macroeconomics (B), Monetary Economics (G), Microeconomics (A), Manpower (L), Industrial Organization (J), and Business Economics (I). Here the "theoretical/applied" descriptor seems to fit quite well. International Economics (H), Comparative Systems (O), Economic History (N) and History of Thought (C), however, seem to be located too high; one might expect them to fall near the x-axis. If the reader is willing to accept this anomaly, the other points seem to be well-located to justify the "theoretical/applied" label.

Turning now to the DREMC respondents, Figure 24 portrays the aggregate configuration for dimension two versus dimension one. Like the OR/AS respondents, the DREMC economists apparently view economic subjects primarily in terms of their quantitative content. Dimension one seems to be a fairly clear "quantitative/non-quantitative" dimension.

The second dimension in Figure 24 appears to be a "micro/macro" scale. With the exception of Economic History (N), and Radical Political Economics (Q), the vertical array of the subjects from top to bottom of the plot seems to be primarily along a continuum of their concern with increasingly more national and international economic issues.

The third dimension, shown in Figure 25, is much more difficult to label. A highly tentative suggestion is "output vs. input orientation" or "process vs. product orientation." Agricultural Economics (K), Regional Economics (P) and Welfare Economics (M) seem to be high in product orientation; that is, they are concerned with the outputs of production and Monetary Economics (G), Comparative Systems (O), and History of Thought (C) seem to be high in process orientation, i.e., concerned with the production process itself. However, the location of subjects such as Economic History (N) and Business Economics (I) seems to argue against this descriptor. Thus, it is not at all clear that this is the best label for dimension three. However, despite a significant effort, nothing better



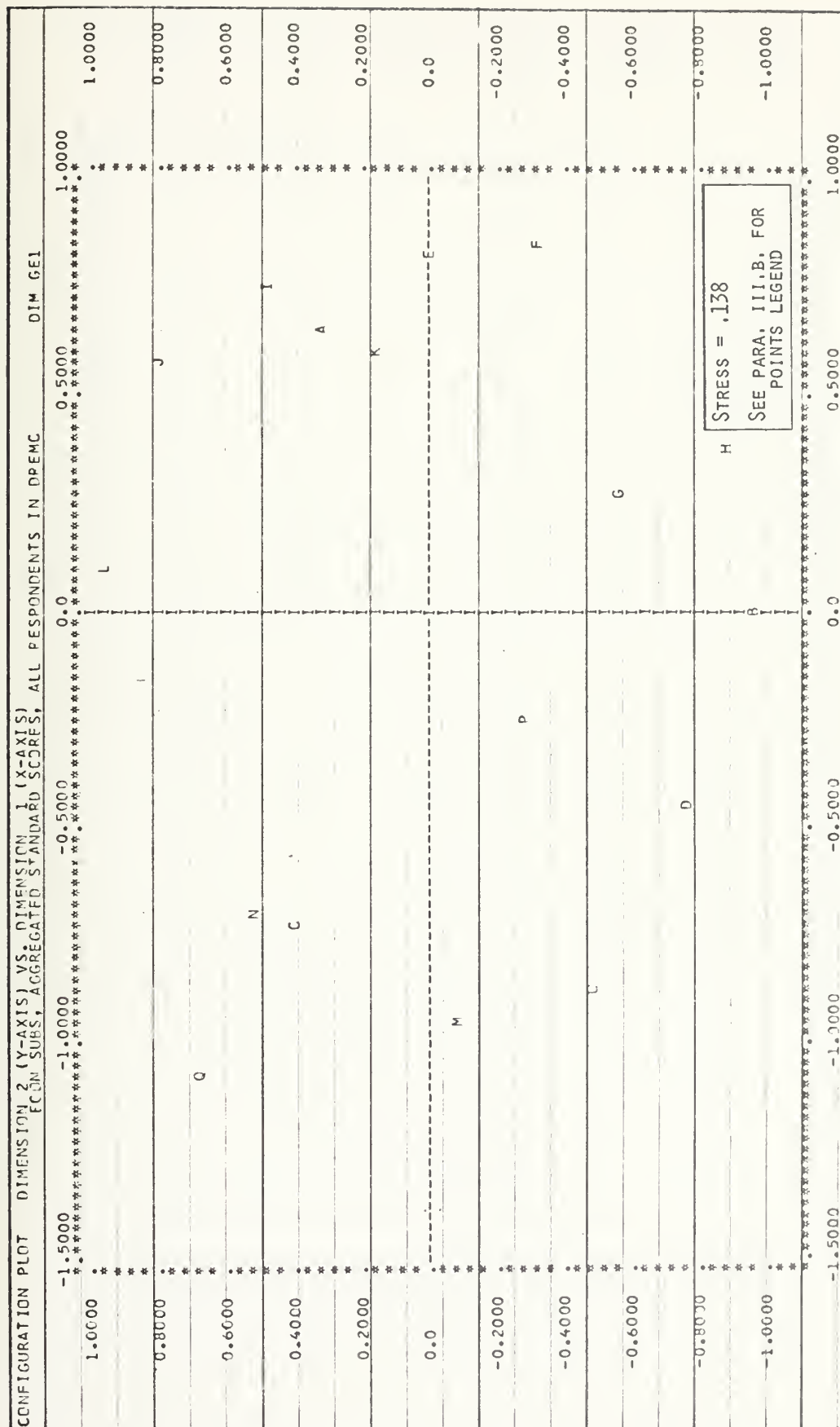


Figure 24



Figure 25

comes to mind, so we will stay with "process/product" as the descriptor.

Finally, looking at the configurations representing the aggregate perceptions of all economists at the Naval Postgraduate School, we can interpret dimension one in Figure 26 fairly readily as a quantitative dimension. However, the second dimension has some difficult groupings. Agricultural Economics (K) and Welfare Economics (M) are quite close; as are Econometrics (F), Regional Economics (P), and History of Thought (C). The arrangement of the subjects in the second dimension seems to defy interpretation. This is not an unexpected result, however, considering that the aggregate results for the two groups of respondents showed no commonly shared second- or third-dimension configurations.

A study of the third dimension configuration in Figure 27 leads to similarly fruitless attempts to assign a descriptor. Econometrics (F), Mathematical Economics (E), and Comparative Systems (O) are grouped together, as are Microeconomics (A), Economic Growth (D), and Radical Political Economics (Q). In short, there seems to be no suitable description of the grouping of the subjects in either the second or third dimension when the perceptions of all economists at the Naval Postgraduate School are aggregated.

To summarize then, all economists at the Naval Postgraduate School apparently perceive economic subjects primarily in terms of their quantitative orientation. However, beyond this primary perception, the results of this study indicate

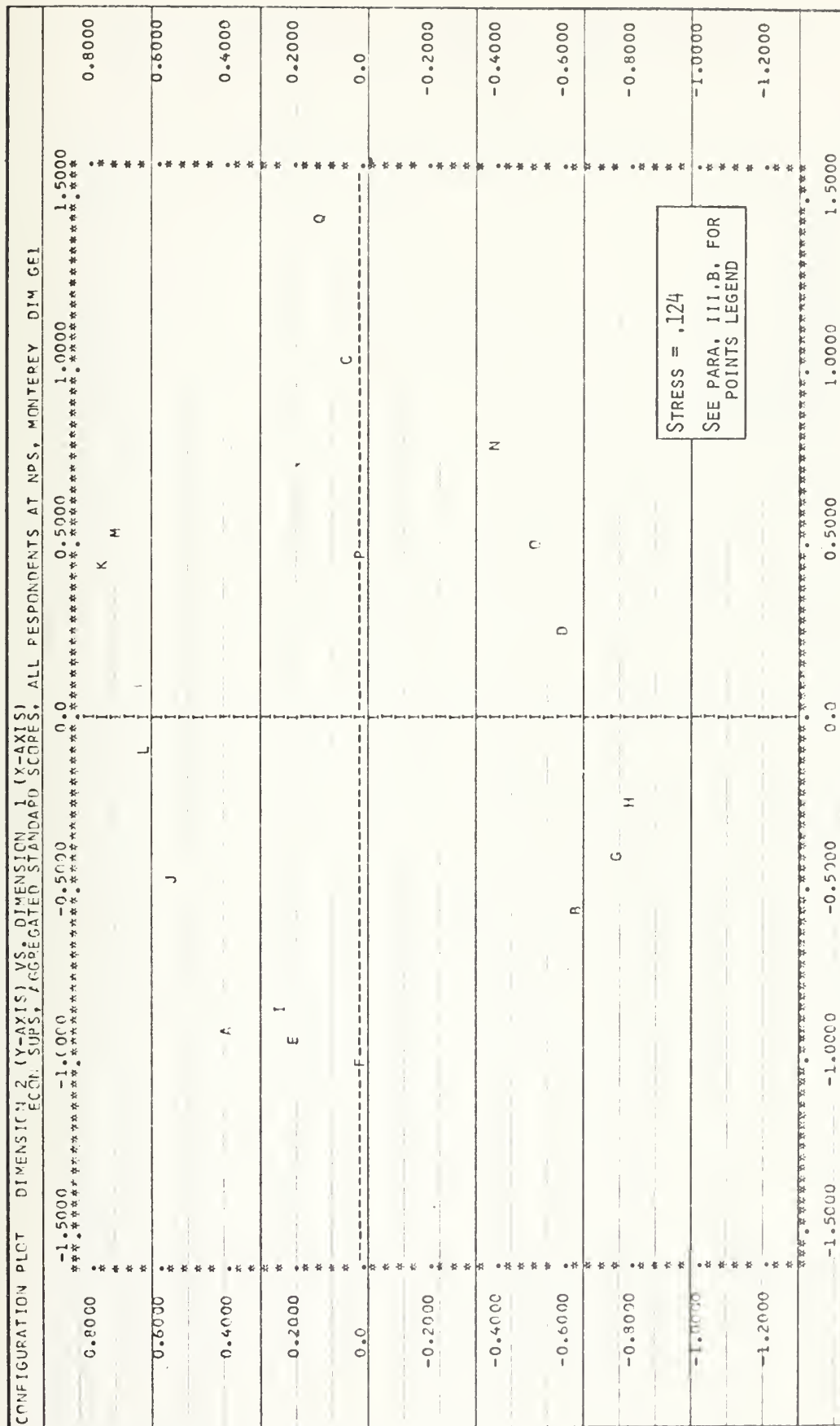


Figure 26



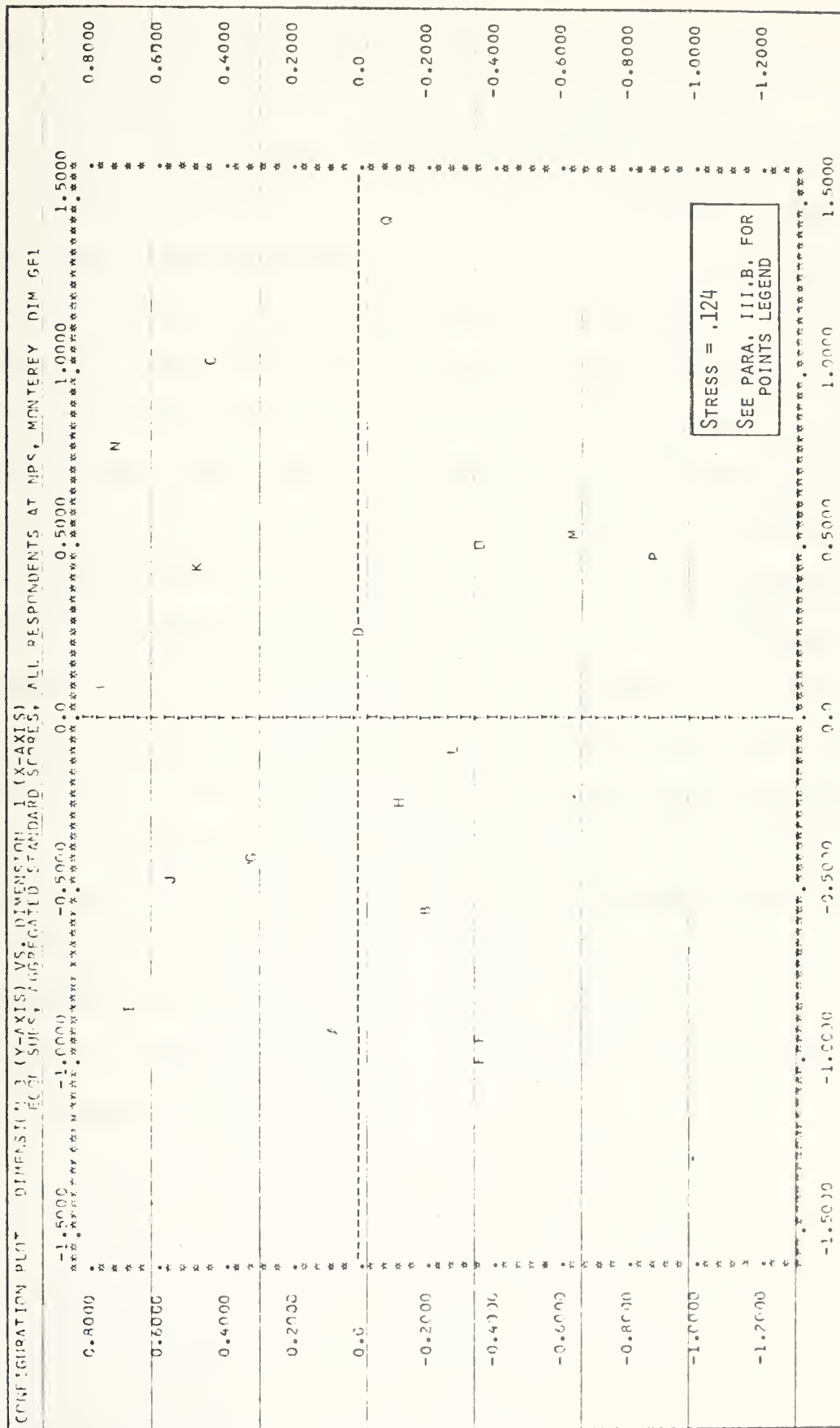


Figure 27

that there are significant differences in the secondary and tertiary perceptions of the OR/AS and the DREMC economists. The second- and third-dimension perceptions of the OR/AS economists were described as "positive/normative" and "theoretical/applied." Both these perceptions seem to be based on the content of the subjects. The DREMC economists' perceptions of the second- and third-dimensions were described as "micro/macro" and "process/product," respectively. Both these descriptors seem to indicate an emphasis on the orientation of the subjects. Or, ot put it a different way, the perceptions of the OR/AS economists seem to be generally analytical, and those of the DREMC economists more descriptive. This interpretation, though admittedly highly subjective, would seem to agree with the orientation of the curriculum taught by each group of economists. One would probably expect to find a more analytical orientation among professors teaching the theoretical aspects of economics in a quantitative curriculum. Likewise, one would expect to find more descriptive, output-oriented perceptions among economists in a curriculum which emphasizes applications of economics.

V. NON-INTERPRETABLE DIMENSIONS -
AN INVESTIGATION AND EXPLANATION

During the process of interviewing the respondents in the study, the question of why some economists could describe all three dimensions as meaningful representations of their perceptions of economic subjects and others could find meaning in only the first dimension seemed to demand some attempt at explanation. While the existence of meaningless dimensions could easily be dismissed as evidence that there was simply nothing meaningful there to portray, some observations made during the interviews led the investigator to question why nothing was there and, for the two respondents who saw a meaningful second-dimension configuration only on the high-interest end of the the first dimension, why only part of something was there. None of the respondents who indicated or stated during the interview that they were unfamiliar with some of the economic subjects was able to fully identify all three dimensions, and both respondents who saw a partially meaningful second dimension indicated that they were not too familiar with some of the subjects in the uninterpretable portion of the second-dimension configuration. Thus a study of the importance of familiarity with the subjects scaled as a determinant of meaningful configurations seemed appropriate. First, however, it is desirable to consider all other plausible reasons for

THE HISTORY OF THE CITY OF BOSTON

FROM THE FIRST SETTLEMENT TO THE PRESENT TIME
BY
JOSEPH NEALE, ESQ.

IN TWO VOLUMES.
THE FIRST VOLUME.

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1796.

THE SECOND VOLUME.

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1796.

THE THIRD VOLUME.

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1796.

the failure of the scaling program to generate meaningful solutions in more than one dimension.

To do this, we must assume that the respondent will name the dimension if he is able to discern it. This would seem to be a reasonable assumption when dealing with economists and economic subjects and there was no evidence that anyone in the test phase was deliberately withholding such information during the interviews. Given this assumption, then a failure to identify the configuration, if it actually is meaningful, could only arise from a gross lack of motivation or simple inability on the part of the respondent to see a relationship where one existed. Neither of these conditions was suspected during this study, and certainly would not be expected of the caliber of persons participating. Therefore we can conclude that the program was actually producing non-meaningful results for some of the respondents.

Invalid results could arise from the program's failure to perform as advertised or from capricious or random rankings. However, the mileage chart and circle recovery tests indicated that the program was performing properly, and capricious or random rankings, if not discovered prior to scaling the data, can be surmised if the configuration is meaningless in the first dimension.

Inconsistent choices are one possible cause of meaningless configurations. For example, if a respondent perceives the pairs in Euclidean space, or in any ρ -metric space

reasonably close to Euclidean, we would expect that if subjects A and B are ranked "1" in similarity, and subjects B and C are ranked "1" in similarity, then subjects A and C would be ranked "1" or "2" in similarity. If this relationship did not hold for a large proportion of the pairs, then the configuration would obviously be degraded. However, analysis of the last sixteen "check questions" for the respondents did not indicate any significant degree of inconsistency in responses. Since previous versions of the scaling program have been shown to exhibit the ability to properly recover the underlying configuration even in the presence of moderate "noise" such as would be caused by inconsistent responses [Green and Carmone, 1970], inconsistent responses can be effectively ruled out as a cause of non-interpretable results among the test phase respondents. A recent study indicates that Minkowski p -metrics for other than Euclidean space are probably applicable only when the subjects scaled are either perceptually distinct and do not interact or when one dominates all others [Sherman, 1972]. Thus, with economic subjects which are perceptually distinct but which are interrelated, we would expect to obtain the best configuration in (or near) Euclidean space.

A. FRAME OF REFERENCE AND PERCEPTUAL ORIENTATION

Since none of the above reasons seems to adequately account for the existence of a meaningless configuration, it can probably be ascribed to changes in the respondent's frame of reference while completing the questionnaire. It

is important to recognize that the term "frame of reference" as used here refers to the primary criteria used by the respondent or his "mental set" while completing the questionnaire. It does not refer to the various perceptual orientations which can exist within this primary criteria or mental set, for it is these different perceptual orientations which translate directly into the different dimensions obtained through a scaling program.

To clarify this somewhat subtle but important distinction, consider again the mileage chart in Section II. The frame of reference used in drawing up the chart was distance and direction. Every entry in the chart was a measure of the similarity of two cities in terms of the great circle distance, in statute miles, between them. If we think of the distance between New York and Boston as having "x" miles of an east component and "y" miles of a north component, it is clear that our perceptual orientation is in terms of an east-west distance and a north-south distance. By using these two components to describe the location of a point we are using two different perceptual orientations, and the resulting configuration will reflect these two orientations by being interpretable in two dimensions. This simple illustration also implies that perceptual orientations are actually subsets of the frame of reference; that is, it is probably not possible to hold perceptual orientations whose components are not included in the frame of reference.

Figure 28 summarizes the expected effect on the configuration obtained through multidimensional scaling of changes in a respondent's frame of reference and changes in his perceptual orientation while performing a pairwise similarity comparison. If the respondent maintains the same frame of reference and does not allow his perceptual orientation to shift, the resulting configuration will be interpretable in only the first dimension, and this dimension will reflect the perceptual orientation held by the respondent as he compared the items to be scaled. This is obvious if we imagine trying to measure intercity distances, for example, only in terms of an east-west perceptual orientation. We could accomplish this quite easily by marking their east-west orientation in degrees of longitude along some line of latitude and then measuring the interpoint distances. However, the only interpretable result of scaling such a one-dimension ranking would be the one-dimension configuration. If scaling in higher dimensions were performed, the results would be meaningful only in the first dimension.

When the respondent holds his frame of reference constant, but allows his perceptual orientation to freely change as he compares the various items being scaled, a multidimensional configuration will be appropriate to describe the respondent's perceptions, and the interpretability or meaningfulness of the various dimensions will reflect how clearly ordered the points are within the respondent's frame of reference.



	Unvarying Perceptions	Changing Perceptual Orientations
Constant Frame of Reference	One Dimension	Interpretable Multi-dimensional Configuration
Changing Frame of Reference	One Dimension	Non-interpretable or only Partially Interpretable Multi-dimensional Configuration

Expected Interpretability of Configurations Resulting from Changes in Frame of Reference and Perceptual Orientation

Figure 28

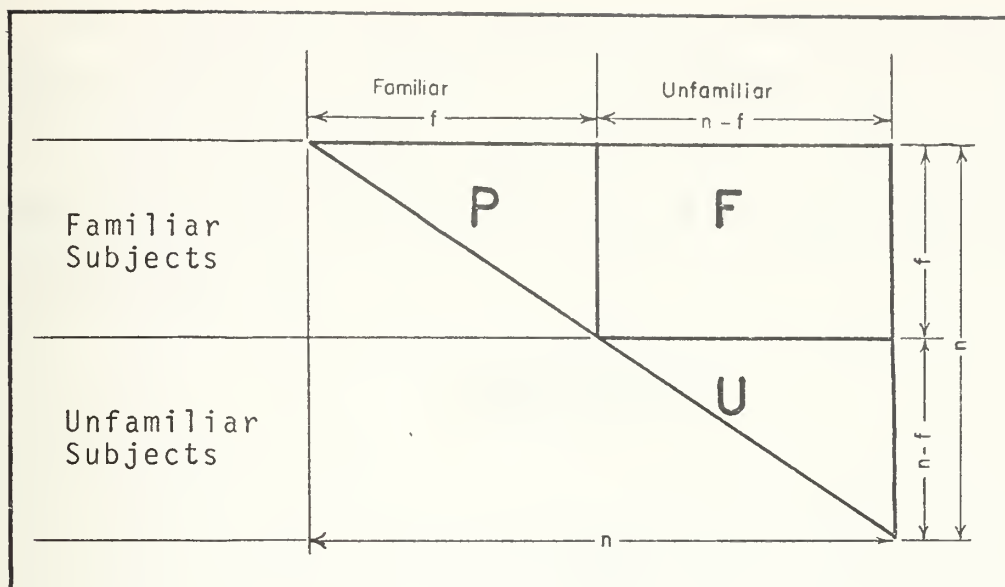
The lower left-hand block of Figure 28 represents an illogical occurrence. Even if one finds different frames of reference in which a single perceptual orientation can be accommodated, they will not affect the final outcome, since holding one's perceptual orientation constant ensures the production of a configuration that is valid in only one dimension. Finally, if one changes both his frame of reference and his perceptual orientation while comparing the similarity of a group of items, the overall configuration will probably be meaningless in dimensions greater than one. For example, if we were to compare some of the cities in terms of distance between them, and the others in terms of their population, we would obviously have a meaningless configuration when viewed as a whole, although some groups of points might display meaningful relationships.

The reader should not infer from the previous discussion that there is a high likelihood of obtaining meaningless multidimensional solutions by the respondent holding his perceptual orientation constant. It is probably difficult, if not impossible to accomplish this feat unless one relies on some mechanical aid such as ordinal ranking the subjects exclusively in terms of how much of the ingredient being measured exists within the single perceptual orientation and then marking the similarity ratings scale for each pair on the basis of the absolute difference between their ordinal ranks.

The previous discussion has been highly general and draws on an easily conceptualized model for its explanation. When one attempts to apply the meaning of "changes in frame of reference" to less concrete examples than mileage charts and maps, it may not be easy to imagine circumstances which would lead one to change his frame of reference while comparing various points, at least if his initial frame of reference is sufficiently broad to logically contain all the perceptual orientations necessary to describe the points. For example, if one is comparing economic subjects based on their "general appeal" to him, that frame of reference is so broad as to include all possible reasons (perceptual orientations) for any subject appealing or not appealing to him. Any change in his frame of reference in this case would only serve to force him to think in more restrictive terms. If we accept as rational human behavior the attempt to express one's perceptions in the most meaningful manner, given sufficient motivation for such expression, then we would not expect a respondent to voluntarily curtail his freedom of choice (especially when the questionnaire is so constructed as to severely curtail his freedom of expression) and select a more restrictive frame of reference in which to express his perceptions of the items being scaled. Therefore, it can now be demonstrated that the most likely explanation for a respondent changing his frame of reference is that he is forced to do so due to unfamiliarity with some of the items presented for comparison.

B. FAMILIARITY AND INTERPRETABILITY

To understand why unfamiliarity with any of the subjects presented for pairwise comparison will force a change to a more restrictive frame of reference, consider an economist who receives the questionnaire in Appendix A which requests him to rate the similarities of each pair in terms of their general appeal to him. His responses can be categorized by means of an upperhalf matrix such as Figure 29. If he is familiar with only "f" of the "n" subjects presented for comparison, and if he attempts to rate each pair in terms of "general appeal," he will be able to rate "P" of the $\frac{1}{2}(n)(n-1)$ pairwise comparisons in terms of "general appeal," as requested. These will represent the pairs of subjects with which he is well conversant. For the "F" pairs of subjects which include a familiar and an unfamiliar subject, the only logical choice he can make is one based on familiarity, since he has no other frame of reference to apply. Thus, when faced with a familiar subject paired with an unfamiliar one, he could be expected to rank them as dissimilar. When he is faced with a pair of unfamiliar subjects, however, his most logical choice would be to rate them as similar, since they both possess similar amounts of the ingredient "unfamiliarity." However, there may well be some natural abhorrence to awarding a "similar" ranking to a pair of unfamiliar subjects. To do so forces the respondent to recognize that his frame of reference is



Effect of Unfamiliarity with Subjects
on Questionnaire Responses

Figure 29

different than when he was ranking two familiar "similar" subjects. Not to do so requires him to rank on some other, probably even less logical basis. Thus the pair of unfamiliar subjects is likely to be disturbing to the respondent, and it is difficult to hypothesize what criteria he could be expected to apply in ranking such pairs. Thus the lower triangle submatrix is labeled "U" since it is unclear what frame of reference the respondent would use in assigning a ranking to the unfamiliar pairs.

The proportion of responses in each category (P, F, and U) can be determined if one knows the number of subjects with which the respondent is unfamiliar. If $n - f > 0$, it can be empirically shown that submatrices P and U will be triangular half-matrices and submatrix F will be a rectangular matrix.

The number of cells in submatrix P, from the usual half-matrix formula is $1/2(f)(f-1)$. To express P as a proportion, f must be expressed as a proportion of n. Thus the proportion of responses which are based on a predetermined frame of reference is:

$$\frac{1/2(f)(f-1)}{1/2(n)(n-1)}$$

or:

$$P = \frac{f^2 - f}{n^2 - n}$$

Similarly, since submatrix U is also an upperhalf matrix, the same formula can be applied, except that n-f is used:

$$U = \frac{1/2(n-f)[(n-f)-1]}{1/2(n)(n-1)}$$

or:

$$U = \frac{(n-f)[(n-f)-1]}{n^2 - n}$$

The remaining rectangular matrix (F), representing the proportion of choices which are based on familiarity rather than preference, has f rows and n-f columns. Its size, proportional to the entire matrix, is:

$$F = \frac{f(n-f)}{1/2(n)(n-1)}$$

or:

$$F = \frac{2(nf - f^2)}{n^2 - n}$$

The above relationships make explicit the importance of familiarity as a determinant of useful responses. For example, to rank even one-half of the 136 pairs in the economics questionnaire in terms of general appeal would require that the respondent be familiar with 13 of the 17 subjects.

Table 7 gives the expected maximum percentage of responses for various numbers of subjects included in a questionnaire (n) and various numbers of subjects with which the respondent is familiar (f). Since, in general, the required proportion of familiar subjects to total subjects presented (f/n) to achieve any specified percentage of preferentially ranked responses increases as the total number of objects scaled decreases, it is clear that familiarity with all of the objects scaled in a study of this size is a necessary prerequisite to totally meaningful configurations in more than one dimension.

It is now possible to better understand the probable meaning of non-interpretable and only partially-interpretable dimensions. It was earlier explained that the subjects with which the respondent is unfamiliar will be arrayed toward the "least-preferred" end of the preference scale, opposite the "preferred" and familiar subjects. If there is more than one unfamiliar subject (regardless of the number scaled) then some of his choices will be "U" type; that is, it is

Number of Objects with which Respondent is Familiar (f)	Number of Objects Scaled (n)			
	12	15	17	20
3	.045	.029	.022	.016
5	.151	.095	.073	.053
6	.227	.143	.110	.079
8	.424	.267	.206	.147
10	.681	.428	.331	.237
12	1.000	.628	.485	.347
15	--	1.000	.772	.553
17	--	--	1.000	.716
18	--	--	--	.805

Expected Percentage of Responses within any
Pre-specified Frame of Reference
for Selected Values

Table 7

unclear how they will be ranked. As the proportion of "U" choices increases, the probability of any meaningful configuration among the unfamiliar subjects decreases. However, since the familiar subjects can be expected to have well-defined perceptual distances, the configuration among them should be meaningful. Thus the existence of a meaningful configuration of points in two or more dimensions toward the "preferred" end of the first dimension scale, coupled with a meaningless configuration toward the least-preferred end can be taken as evidence that the respondent was unfamiliar with some of the subjects presented for comparison.

Similarly, the existence of uninterpretable dimensions higher than one can be taken as evidence that the respondent was unfamiliar with a large proportion of the subjects presented for comparison. Unless the subjects presented are so sterile as to allow comparison in terms of only one perceptual orientation, the rational respondent can be expected to call on various perceptual orientations while ranking the subjects, given that he is sufficiently familiar with them to enable comparison in terms of more than one perceptual orientation. Since it is these different perceptual orientations which correspond to the various dimensions obtained through the scaling program, meaningless multidimensional configurations strongly suggest a lack of familiarity with many of the subjects presented for comparison.

C. A POTENTIAL APPLICATION OF MULTIDIMENSIONAL SCALING

It is now possible to summarize the significance of this investigation into the meaning of noninterpretable results and to suggest a potentially useful application of multidimensional scaling to a field in which it has heretofore not been applied. Since noninterpretable configurations in dimensions higher than one strongly indicate a lack of understanding of the content of some of the items scaled, if the items scaled represent items with which the respondent should have some depth of knowledge, and if knowledgeability of the content of these items is considered to be a necessary prerequisite for successful performance in the respondent's occupation, then non-interpretable dimensions higher than one can be taken as a fairly clear indication that the respondent's performance in that field will be less than optimal. Thus multidimensional scaling would seem to be an excellent medium for evaluating the professional performance of persons whose occupations require a depth of knowledge in various complex subjects.

At this point it is necessary that the reader understand that the results of this study into economists' perceptions of economic subjects should not be taken as evidence of the professional performance of the respondents. There was not any attempt, in selecting the seventeen subjects scaled, to utilize only those in which a high degree of knowledgeability would be expected from the respondents. For example, it seems unlikely that a depth of understanding of Agricultural



Economics or Radical Political Economics is at all important to the performance of an economist teaching in a curriculum oriented to the needs of middle and high level managers in the Department of Defense. To accurately assess the performance of any group of professional persons, the subjects utilized must be carefully selected to include only those with which the respondent is expected to have a depth of understanding. Thus the reader is cautioned not to infer any relationship between the results of this study and the professional performance of the respondents without first carefully assessing the significance of the subjects scaled to the school's requirements for knowledgeability in these subjects.

Obviously, the potential application of multidimensional scaling as a tool for evaluating the performance of professional persons is limited only by the ability of the evaluator to determine a sufficient number of applicable topics or subjects for pairwise comparison. It offers the promise of obtaining a meaningful evaluation based on the most important determinant of professional performance, the respondent's knowledgeability in the various aspects of his profession. Furthermore, it allows the rater to rank the respondent's performance in terms of the respondent's absolute degree of understanding of the "tools" of his profession, not in comparison with his peers and unbiased by personal opinion, political considerations, or the "halo effect."

VI. SUMMARY OF RESULTS

The important findings of this study can now be summarized within three categories. Technical findings are those empirically determined findings which relate to the operation of the scaling program itself under different input control parameters. Descriptive findings are observations related to scaling and interpreting the configurations for the respondents, both individually and after aggregation. Analytical findings are those stemming from an analysis of the meaning of non-interpretable dimensions.

A. TECHNICAL FINDINGS

1. Both raw scores and standardized scores lead to essentially identical configurations for individual respondents when the input data are obtained from ordinal similarity measures of pairwise comparisons.

2. The "primary" (unconstrained) approach to ties is appropriate when the input data are non-metric and represent coarse category judgments.

3. Only data aggregations based on standard scores can be expected to yield meaningful configurations.

4. When random starting configurations are used, initial stress is not related to final stress.

5. The TORSCA initial configuration used in the KYST scaling program can be expected to lead to the lowest



stress solution (global minimum) in dimensions two and three. Random starting configurations may result in lower final stress in dimension one, but often terminate at local minimums in dimensions two and three. (Dimensions higher than three were not tested.)

B. DESCRIPTIVE FINDINGS

1. The prior impressions of economic subjects obtained in graduate school may have a long-term effect on an economist's perceptions of those subjects.

2. The relative positioning of economic subjects in which the economist currently has a high interest will be amplified in the scaling configuration.

3. A scaling configuration is completely valid only for the point in time in which the responses were generated and reflects short-term, possibly transitory perceptions as well as long-run, deeply-held perceptions.

4. Economists at both the Defense Resource Management Center and in the OR/AS Department apparently perceive economic subjects primarily in terms of their quantitative orientation.

5. The secondary and tertiary perceptions of economic subjects by economists in the OR/AS Department are apparently most influenced by the theoretical underpinnings of the subjects.

6. The secondary and tertiary perceptions of economic subjects by economists at the Defense Resource Management

Center are apparently most influenced by the descriptive and operational aspects of the subjects.

C. ANALYTICAL FINDINGS

1. If the first dimension is "preference" or "interest," unfamiliar subjects will tend to appear toward the low-preference or low-interest end of the configuration.

2. The existence of one or more dimensions higher than one which are interpretable only in the "high-interest" or "high-preference" direction indicates unfamiliarity with some of the subjects scaled.

3. The existence of non-interpretable dimensions higher than one indicates a lack of familiarity with or knowledgeability in the subjects scaled.

4. The existence of logically interpretable dimensions higher than one indicates familiarity with and understanding of the subjects scaled.

5. Multidimensional scaling appears to be well-suited as a tool for evaluating the performance of professional persons where effective job performance is predicated upon knowledgeability in many complex subjects.

APPENDIX A

The following letter and questionnaire were sent to economists at the Naval Postgraduate School and the Defense Resource Management Center.

Naval Postgraduate School
SMC 1511
Monterey, CA 93940
23 April 1975

Dear Professor:

As a part of my thesis requirements, I am assisting in a study of economists' perceptions of economic subjects. The enclosed questionnaire is one way by which perceptions can be described. Will you please complete it in one sitting (it should take 30 to 45 minutes) and return it to me in the enclosed pre-addressed envelope? If you will sign the instruction sheet of the questionnaire, I will return the "computer map" of your perceptions so you can see the computer's representation of your interests. Of course, you may remain anonymous, however from the economists who have already participated, I think you will find the results quite interesting.

Charles D. Gee
LCDR, SC, USN

Enclosure

Thesis Advisor: Prof. G. L. Musgrave, IN-201, ext. 2052

INSTRUCTIONS

Please rate how similar or dissimilar the economics subjects named in each pair are in their general interest to you by circling the number on the continuum that best represents your feelings about the general appeal of these two subjects. Do not leave any questions blank.

	1	2	3	4	5	6	7	8	9
Econometrics	Exactly								
Business Economics	Equal				Extremely Different				

a. If Econometrics and Business Economics are exactly equal in their general appeal to you, you should circle the number one (1) in the row of numbers by the two economics subjects.

b. If Econometrics and Business Economics are extremely different in their general appeal to you, you should circle the number nine (9) in the row of numbers.

c. If there is a difference between Econometrics and Business Economics in their appeal to you, but you feel a nine (9) overstates the difference and a one (1) understates the difference, you should circle a number between one (1) and nine (9) on the continuum that best represents the difference you feel exists between the general appeal of Econometrics and Business Economics.

DO NOT LEAVE ANY QUESTIONS BLANK.

	1	2	3	4	5	6	7	8	9
1. Econometrics Business Economics	Exactly Equal								Extremely Different
2. Manpower & Labor Economics Radical Political Economics	1	2	3	4	5	6	7	8	9
3. Macroeconomics History of Economic Thought	1	2	3	4	5	6	7	8	9
4. Economic Growth & Development Comparative Economic Systems	1	2	3	4	5	6	7	8	9
5. Microeconomics History of Economic Thought	1	2	3	4	5	6	7	8	9
6. Macroeconomics Mathematical Economics	1	2	3	4	5	6	7	8	9
7. Agricultural Economics Economic History	1	2	3	4	5	6	7	8	9
8. Macroeconomics Economic Growth & Development	1	2	3	4	5	6	7	8	9
9. Economic History Comparative Economic Systems	1	2	3	4	5	6	7	8	9
10. Comparative Economic Systems Radical Political Economics	1	2	3	4	5	6	7	8	9
11. Microeconomics Industrial Organization	1	2	3	4	5	6	7	8	9
12. Comparative Economic Systems Regional Economics	1	2	3	4	5	6	7	8	9
13. Business Economics Agricultural Economics	1	2	3	4	5	6	7	8	9
14. Microeconomics Radical Political Economics	1	2	3	4	5	6	7	8	9
15. Economic Growth & Development {Welfare Programs, Consumer Economics & Urban Economics	1	2	3	4	5	6	7	8	9
16. International Economics {Welfare Programs, Consumer Economics & Urban Economics	1	2	3	4	5	6	7	8	9

		1	2	3	4	5	6	7	8	9
17.	Business Economics Regional Economics	Exactly Equal								Extremely Different
18.	History of Economic Thought Economic History	1	2	3	4	5	6	7	8	9
19.	Macroeconomics Manpower & Labor Economics	1	2	3	4	5	6	7	8	9
20.	Macroeconomics Industrial Organization	1	2	3	4	5	6	7	8	9
21.	Econometrics International Economics	1	2	3	4	5	6	7	8	9
22.	Monetary Economics Economic History	1	2	3	4	5	6	7	8	9
23.	Manpower & Labor Economics {Welfare Programs, Consumer Economics & Urban Economics	1	2	3	4	5	6	7	8	9
24.	Economic Growth & Development International Economics	1	2	3	4	5	6	7	8	9
25.	Industrial Organization {Welfare Programs, Consumer Economics & Urban Economics	1	2	3	4	5	6	7	8	9
26.	History of Economic Thought Industrial Organization	1	2	3	4	5	6	7	8	9
27.	History of Economic Thought International Economics	1	2	3	4	5	6	7	8	9
28.	Agricultural Economics Comparative Economic Systems	1	2	3	4	5	6	7	8	9
29.	History of Economic Thought Regional Economics	1	2	3	4	5	6	7	8	9
30.	History of Economic Thought Mathematical Economics	1	2	3	4	5	6	7	8	9
31.	International Economics Agricultural Economics	1	2	3	4	5	6	7	8	9
32.	History of Economic Thought Economic Growth & Development	1	2	3	4	5	6	7	8	9
33.	Economic Growth & Development Radical Political Economics	1	2	3	4	5	6	7	8	9

	1	2	3	4	5	6	7	8	9
34. Monetary Economics {Welfare Programs, Consumer Economics & Urban Economics	Exactly Equal								Extremely Different
35. Microeconomics Macroeconomics	1	2	3	4	5	6	7	8	9
36. Econometrics Economic History	1	2	3	4	5	6	7	8	9
37. Business Economics {Welfare Programs, Consumer Economics & Urban Economics	1	2	3	4	5	6	7	8	9
38. History of Economic Thought Econometrics	1	2	3	4	5	6	7	8	9
39. Econometrics {Welfare Programs, Consumer Economics & Urban Economics	1	2	3	4	5	6	7	8	9
40. Econometrics Manpower & Labor Economics	1	2	3	4	5	6	7	8	9
41. Mathematical Economics {Welfare Programs, Consumer Economics & Urban Economics	1	2	3	4	5	6	7	8	9
42. Business Economics Radical Political Economics	1	2	3	4	5	6	7	8	9
43. International Economics Economic History	1	2	3	4	5	6	7	8	9
44. Business Economics Comparative Economic Systems	1	2	3	4	5	6	7	8	9
45. International Economics Manpower & Labor Economics	1	2	3	4	5	6	7	8	9
46. Business Economics Economic History	1	2	3	4	5	6	7	8	9
47. Agricultural Economics Regional Economics	1	2	3	4	5	6	7	8	9
48. Macroeconomics Monetary Economics	1	2	3	4	5	6	7	8	9
49. Econometrics Industrial Organization	1	2	3	4	5	6	7	8	9

	1	2	3	4	5	6	7	8	9
50. Manpower & Labor Economics Economic History	Exactly Equal								Extremely Different
51. { Welfare Programs, Consumer Economics & Urban Economics Radical Political Economics	1	2	3	4	5	6	7	8	9
52. Mathematical Economics Econometrics	1	2	3	4	5	6	7	8	9
53. Monetary Economics International Economics	1	2	3	4	5	6	7	8	9
54. History of Economic Thought Radical Political Economics	1	2	3	4	5	6	7	8	9
55. Econometrics Radical Political Economics	1	2	3	4	5	6	7	8	9
56. International Economics Radical Political Economics	1	2	3	4	5	6	7	8	9
57. Industrial Organization Manpower & Labor Economics	1	2	3	4	5	6	7	8	9
58. Monetary Economics Business Economics	1	2	3	4	5	6	7	8	9
59. Industrial Organization Agricultural Economics	1	2	3	4	5	6	7	8	9
60. International Economics Regional Economics	1	2	3	4	5	6	7	8	9
61. Econometrics Monetary Economics	1	2	3	4	5	6	7	8	9
62. Monetary Economics Radical Political Economics	1	2	3	4	5	6	7	8	9
63. Microeconomics Regional Economics	1	2	3	4	5	6	7	8	9
64. Economic Growth & Development Monetary Economics	1	2	3	4	5	6	7	8	9
65. { Welfare Programs, Consumer Economics & Urban Economics Regional economics	1	2	3	4	5	6	7	8	9
66. Monetary Economics Comparative Economic Systems	1	2	3	4	5	6	7	8	9

	1	2	3	4	5	6	7	8	9
67. Microeconomics Business Economics	Exactly Equal								Extremely Different
68. Mathematical Economics Economic History	1	2	3	4	5	6	7	8	9
69. Economic Growth & Development Regional Economics	1	2	3	4	5	6	7	8	9
70. Microeconomics Economic History	1	2	3	4	5	6	7	8	9
71. Microeconomics {Welfare Programs, Consumer Economics & Urban Economics	1	2	3	4	5	6	7	8	9
72. Mathematical Economics Manpower & Labor Economics	1	2	3	4	5	6	7	8	9
73. Mathematical Economics Industrial Organization	1	2	3	4	5	6	7	8	9
74. Manpower & Labor Economics Regional Economics	1	2	3	4	5	6	7	8	9
75. Macroeconomics Agricultural Economics	1	2	3	4	5	6	7	8	9
76. Monetary Economics Agricultural Economics	1	2	3	4	5	6	7	8	9
77. International Economics Business Economics	1	2	3	4	5	6	7	8	9
78. Mathematical Economics Business Economics	1	2	3	4	5	6	7	8	9
79. Mathematical Economics Monetary Economics	1	2	3	4	5	6	7	8	9
80. History of Economic Thought Agricultural Economics	1	2	3	4	5	6	7	8	9
81. Industrial Organization Economic History	1	2	3	4	5	6	7	8	9
82. Macroeconomics International Economics	1	2	3	4	5	6	7	8	9
83. Microeconomics Agricultural Economics	1	2	3	4	5	6	7	8	9

	1	2	3	4	5	6	7	8	9
84. Agricultural Economics {Welfare Programs, Consumer Economics & Urban Economics	Exactly Equal								Extremely Different
85. Business Economics Manpower & Labor Economics	1	2	3	4	5	6	7	8	9
86. Microeconomics Manpower & Labor Economics	1	2	3	4	5	6	7	8	9
87. Monetary Economics Regional Economics	1	2	3	4	5	6	7	8	9
88. Econometrics Regional Economics	1	2	3	4	5	6	7	8	9
89. Economic Growth & Development Economic History	1	2	3	4	5	6	7	8	9
90. Econometrics Agricultural Economics	1	2	3	4	5	6	7	8	9
91. {Welfare Programs, Consumer Economics & Urban Economics Comparative Economic Systems	1	2	3	4	5	6	7	8	9
92. Economic Growth & Development Manpower & Labor Economics	1	2	3	4	5	6	7	8	9
93. Economic Growth & Development Agricultural Economics	1	2	3	4	5	6	7	8	9
94. Mathematical Economics Agricultural Economics	1	2	3	4	5	6	7	8	9
95. Macroeconomics {Welfare Programs, Consumer Economics & Urban Economics	1	2	3	4	5	6	7	8	9
96. International Economics Comparative Economic Systems	1	2	3	4	5	6	7	8	9
97. Macroeconomics Regional Economics	1	2	3	4	5	6	7	8	9
98. Monetary Economics Manpower & Labor Economics	1	2	3	4	5	6	7	8	9
99. Business Economics Industrial Organization	1	2	3	4	5	6	7	8	9
100. Industrial Organization Radical Political Economics	1	2	3	4	5	6	7	8	9

		1	2	3	4	5	6	7	8	9
101.	Regional Economics Radical Political Economics	Exactly Equal								Extremely Different
102.	Microeconomics International Economics	1	2	3	4	5	6	7	8	9
103.	Macroeconomics Radical Political Economics	1	2	3	4	5	6	7	8	9
104.	Microeconomics Comparative Economic Systems	1	2	3	4	5	6	7	8	9
105.	Macroeconomics Business Economics	1	2	3	4	5	6	7	8	9
106.	International Economics Industrial Organization	1	2	3	4	5	6	7	8	9
107.	Economic Growth & Development Econometrics	1	2	3	4	5	6	7	8	9
108.	{ Welfare Programs, Consumer Economics & Urban Economics Economic History	1	2	3	4	5	6	7	8	9
109.	Economic Growth & Development Mathematical Economics	1	2	3	4	5	6	7	8	9
110.	Microeconomics Econometrics	1	2	3	4	5	6	7	8	9
111.	Mathematical Economics Radical Political Economics	1	2	3	4	5	6	7	8	9
112.	Economic Growth & Development Business Economics	1	2	3	4	5	6	7	8	9
113.	Agricultural Economics Radical Political Economics	1	2	3	4	5	6	7	8	9
114.	History of Economic Thought Manpower & Labor Economics	1	2	3	4	5	6	7	8	9
115.	Macroeconomics Econometrics	1	2	3	4	5	6	7	8	9
116.	Economic Growth & Development Industrial Organization	1	2	3	4	5	6	7	8	9
117.	Mathematical Economics Regional Economics	1	2	3	4	5	6	7	8	9
118.	Mathematical Economics Comparative Economic Systems	1	2	3	4	5	6	7	8	9

	1	2	3	4	5	6	7	8	9
119. Microeconomics Economic Growth & Development	Exactly Equal								Extremely Different
120. Economic History Radical Political Economics	1	2	3	4	5	6	7	8	9
121. History of Economic Thought Monetary Economics	1	2	3	4	5	6	7	8	9
122. Mathematical Economics International Economics	1	2	3	4	5	6	7	8	9
123. Agricultural Economics Manpower & Labor Economics	1	2	3	4	5	6	7	8	9
124. Economic History Regional Economics	1	2	3	4	5	6	7	8	9
125. Industrial Organization Regional Economics	1	2	3	4	5	6	7	8	9
126. History of Economic Thought Business Economics	1	2	3	4	5	6	7	8	9
127. Microeconomics Mathematical Economics	1	2	3	4	5	6	7	8	9
128. Industrial Organization Comparative Economic Systems	1	2	3	4	5	6	7	8	9
129. Macroeconomics Comparative Economic Systems	1	2	3	4	5	6	7	8	9
130. Macroeconomics Economic History	1	2	3	4	5	6	7	8	9
131. Econometrics Comparative Economic Systems	1	2	3	4	5	6	7	8	9
132. History of Economic Thought Comparative Economic Systems	1	2	3	4	5	6	7	8	9
133. Monetary Economics Industrial Organization	1	2	3	4	5	6	7	8	9
134. Manpower & Labor Economics Comparative Economic Systems	1	2	3	4	5	6	7	8	9
135. History of Economic Thought { Welfare Programs, Consumer Economics & Urban Economics	1	2	3	4	5	6	7	8	9
136. Microeconomics Monetary Economics	1	2	3	4	5	6	7	8	9

		1	2	3	4	5	6	7	8	9
137.	Microeconomics Radical Political Economics	Exactly Equal								Extremely Different
138.	Macroeconomics History of Economic Thought	1	2	3	4	5	6	7	8	9
139.	International Economics Agricultural Economics	1	2	3	4	5	6	7	8	9
140.	Econometrics International Economics	1	2	3	4	5	6	7	8	9
141.	Business Economics { Welfare Programs, Consumer Economics & Urban Economics	1	2	3	4	5	6	7	8	9
142.	Macroeconomics Monetary Economics	1	2	3	4	5	6	7	8	9
143.	History of Economic Thought Radical Political Economics	1	2	3	4	5	6	7	8	9
144.	International Economics Regional Economics	1	2	3	4	5	6	7	8	9
145.	Industrial Organization Economic History	1	2	3	4	5	6	7	8	9
146.	Microeconomics Business Economics	1	2	3	4	5	6	7	8	9
147.	Econometrics Agricultural Economics	1	2	3	4	5	6	7	8	9
148.	Economic Growth & Development Economic History	1	2	3	4	5	6	7	8	9
149.	Macroeconomics Econometrics	1	2	3	4	5	6	7	8	9
150.	International Economics Industrial Organization	1	2	3	4	5	6	7	8	9
151.	Macroeconomics Economic History	1	2	3	4	5	6	7	8	9
152.	Microeconomics Mathematical Economics	1	2	3	4	5	6	7	8	9

APPENDIX B

STANTRIX PROGRAM DESCRIPTION

GENERAL

This program accepts upperhalf matrices of dissimilarities data and converts them into upperhalf matrices of adjusted standard scores to be used as input data into a scaling program. The user may also request that all individual matrices be aggregated by three different methods. Output consists of a printout of the standardized matrices and, if requested, three aggregate matrices. The user may obtain punched card output for each matrix for use as input to a multidimensional scaling program. One other option allows the user to specify a constant to be added to each standardized matrix cell to prevent negative distances from occurring in the output.

PROGRAM OPERATION

The flow chart of this program is shown in Figure B-1. Its operation can be summarized as follows. For each of m upperhalf matrices, one for each respondent $\ell, \ell+1, \ell+2, \dots, m-1, m$, first compute the mean of all matrix cell values where k_i =number of objects scaled, $k = k_i - 1$ or the number of rows and maximum number of columns in the matrix, and x_{ij} represents the initial value of cell ij in the matrix, as follows:

$$\left. \frac{\sum_{i=1}^{i=k} \sum_{j=1}^{j=ki-i} x_{ij}^{(\ell)}}{(ki)(k)/2} = \bar{X}_{\ell} \right|_{\ell=1, \dots, m} \quad (1)$$

Then compute the standard deviation (S) of each matrix for respondents 1 through m as follows:

$$\left[\frac{\sum_{i=1}^{i=k} \sum_{j=1}^{j=ki-i} (x_{ij}^{(\ell)})^2 - \frac{\left(\sum_{i=1}^{i=k} \sum_{j=1}^{j=ki-i} x_{ij}^{(\ell)} \right)^2}{(ki)(k)/2}}{[(ki)(k)/2] - 1} \right]^{1/2} = S_{(\ell)} \quad \left|_{\ell=1, \dots, m} \quad (2)\right.$$

Then transform each matrix cell (x_{ij}) to a standard score (z_{ij}) by the formula:

$$z_{ij}^{(\ell)} = \frac{x_{ij}^{(\ell)} - \bar{X}_{\ell}}{S_{(\ell)}} \quad \left| \begin{array}{l} i=1, \dots, k \\ j=1, \dots, ki-1 \end{array} \right|_{\ell=1, \dots, m} \quad (3)$$

Then add a constant (K) to each standardized matrix cell (z_{ij}) to prevent negative distances from occurring in the final individual matrix which will be used as input into the

scaling program. Call each matrix cell so adjusted az_{ij} . (If the final matrix is not to be used as input to a scaling program and the user desires to obtain only standard scores, the constant 0 may be used):

$$az_{ij} = z_{ij} + K \quad \left| \begin{array}{l} i=1, \dots, k \\ j=1, \dots, (ki-i) \end{array} \right. \quad (4)$$

If individual matrices are to be aggregated, three different methods of aggregation are used to produce three different aggregate matrices. The first aggregate matrix produced (labeled AGNEW in the program) represents simply the averaged unadjusted standard scores in each cell across all individual standardized matrices (from 1 to m) to which a constant (K) is added:

$$AGNEW_{(ij)} = \frac{\sum_{\ell=1}^m z_{ij}^{(\ell)}}{m} + K \quad \left| \begin{array}{l} i=1, \dots, k \\ j=1, \dots, (ki-i) \end{array} \right. \quad (5)$$

The second aggregate matrix (AGRAW) is developed by first summing and averaging the original data (the raw scores) in each cell across all individual original raw score matrices:

$$\bar{X}_{AGRAW(ij)} = \frac{\sum_{\ell=1}^m x_{ij}^{(\ell)}}{\quad} \quad \left| \begin{array}{l} i=1, \dots, k \\ j=1, \dots, (k_i-i) \end{array} \right. \quad (6)$$

The mean (\bar{X}_{AGRAW}) and standard deviation (S_{AGRAW}) of this aggregate matrix is then computed by means of formulas (1) and (2) above (substituting $\bar{X}_{AGRAW(ij)}$ for x_{ij} and setting $\ell = 1$). The average raw scores in the aggregate matrix AGRW are then transformed into standardized scores by means of formula (3) above (with the same substitutions as before). The final cell values in matrix AGRW are then computed by adding the matrix mean (which is also the mean of all individual original responses) to each standardized cell value:

$$AGRAW(ij) = \frac{\bar{X}_{AGRAW(ij)} - \bar{X}_{AGRAW}}{S_{AGRAW}} + \bar{X}_{AGRAW} \quad \left| \begin{array}{l} i=1, \dots, k \\ j=1, \dots, (k_i-i) \end{array} \right. \quad (7)$$

The final aggregate matrix (ADSTD) is developed by first summing and averaging the standardized scores (after adjustment through the addition of a constant) across all individual matrices:

$$\bar{X}_{ADSTD}(ij) = \frac{\sum_{\ell=1}^m az_{ij}^{(\ell)}}{m} \quad \left| \begin{array}{l} i=1, \dots, k \\ j=1, \dots, (ki-i) \end{array} \right. \quad (8)$$

The mean (\bar{X}_{ADSTD}) and standard deviation (S_{ADSTD}) of this aggregate matrix are then computed by formulas (1) and (2) above (substituting $\bar{X}_{ADSTD}(ij)$ for x_{ij} and setting $\ell = 1$). The average adjusted standard scores in the aggregate matrix are then standardized by use of formula (3) with substitutions as before. The aggregate matrix mean (\bar{X}_{ADSTD} , which should equal, or be very close to, the constant added in the individual computations) is then added to each cell value in the matrix to eliminate negative numbers:

$$ADSTD(ij) = \frac{\bar{X}_{ADSTD}(ij) - \bar{X}_{ADSTD}}{S_{ADSTD}} + \bar{X}_{ADSTD} \quad \left| \begin{array}{l} i=1, \dots, k \\ j=ki-i \end{array} \right. \quad (9)$$

PROGRAM INPUT

Required input data to the program STANTRIX consists of the object deck with card 100 describing the format (in floating decimal format) of the input data, and a data deck assembled as follows:

Parameter Card

Header Card to introduce data of respondent 1

Data for respondent 1

Header Card to introduce data of respondent 2

Data for respondent 2

.
.
.

Header Card for respondent m

Data for respondent m

The parameter card is prepared as shown in Figure B-2.

Some amplifying comments follow:

Number of Objects Scaled

This will always equal the number of matrix rows + 1, since the diagonal matrix row is, by definition, zero in a symmetrical halfmatrix such as is used in this type of scaling.

Number of Subjects

Enter the number of respondents whose individual matrices are being standardized.

Constant

Enter the constant which is to be added to the standard scores in each matrix cell. The constant 0.000 is acceptable. The decimal point should be punched in column 11 and entries made in column 12-14. It is not necessary to zero fill to the left of the leading integer.

Punch Suppress Code

Enter a zero in column 6 if no punched card output is desired. Enter a 1 if punched cards for use as input to a scaling program are desired. Punched cards are produced in format 10F8.3, unless a different format is specified by the user by changing card 300 in the object deck.

Aggregate Suppress Code

Punching a zero in column 18 will produce only individual adjusted standardized matrices. If the three aggregate matrices are desired as well as one for each individual respondent, punch a 1 in this column.

Header Card

The header card consists simply of the respondent's number punched in columns 1-4.

PROGRAM OUTPUT

If the punch and aggregate options are specified the program produces the following output:

For each individual respondent:

Printed Output:

- the respondent's number
- the sum of all matrix cells
- the squared sum of all matrix cells
- the raw matrix mean
- the raw matrix standard deviation
- the adjusted standardized matrix

Card Output:

- header card showing respondent's number to introduce output deck
- data deck of adjusted standardized values

For Aggregated Respondents:

Printed Output:

- the mean of all individual cell values in all input matrices
- for AGRW and ADSTD:
 - the sum of the aggregate matrix cells (before standardizing)
 - the standard deviation of the aggregate matrix cells (before standardizing)
 - the aggregate matrix mean (before standardizing)
 - the aggregate matrix standard deviation (before standardizing)
- the adjusted, standardized aggregate matrix

Card Output:

- header card to introduce the matrix
- data deck of the aggregate matrix

STANTRIX Flowchart

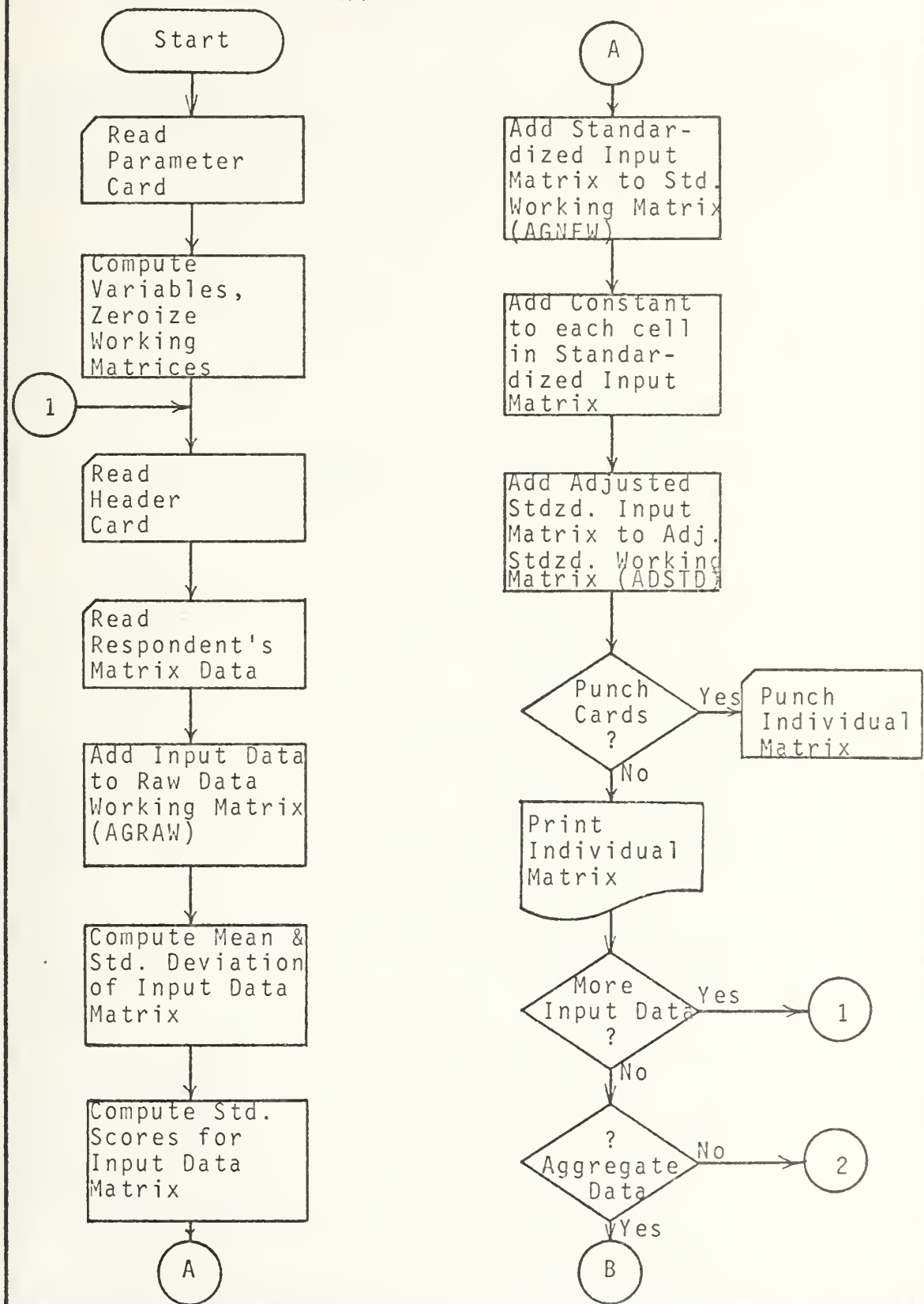


Figure B-1

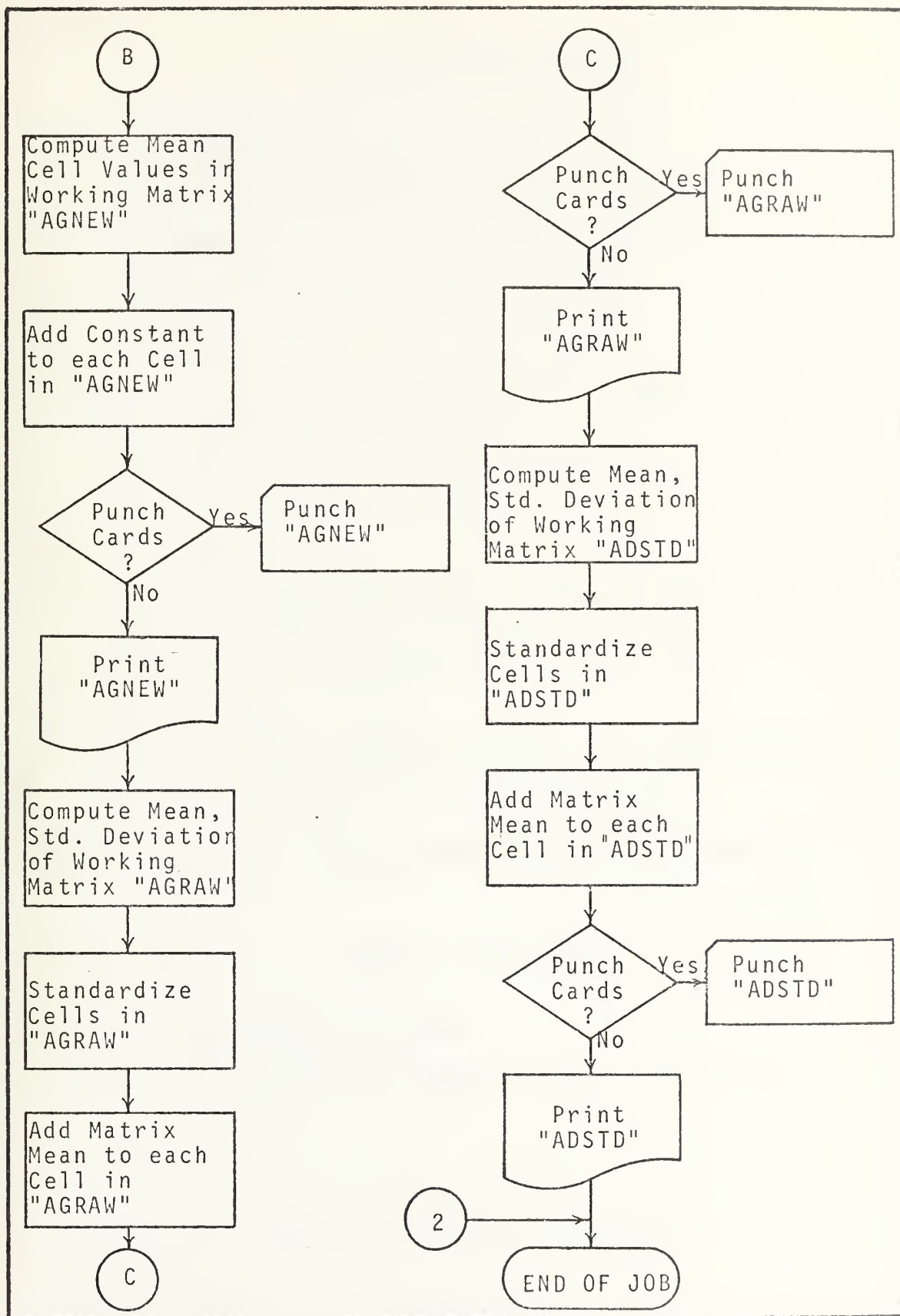


Figure B-1 (continued)

PARAMETER CARD FORMAT

<u>c/c</u>	<u>Entry</u>
1-2	Number of Objects (Stimuli) scaled
3-5	Number of Subjects (respondents) whose data decks are to be included in the run
6	Blank
7-14	Value of the constant to be added to each standard score. Format is F9.3. Decimal should be punched in c/c 11.
15	Blank
16	Punch Suppress Code. 0 (no punched card output) or 1 (punched card output desired).
17	Blank
18	Aggregate Suppress Code. 0 (no aggregated output) or 1 (aggregated output desired).
19-80	Blank or User Comments (not reproduced).

HEADER CARD FORMAT

1-4	Respondent's Number
5-80	Blank or User Comments (not reproduced)

STANTRIX CONTROL CARD FORMATS

Figure B-2

STANTRIX

```
// EXEC FORTCLG
//FORT.SYSIN DD *
  DIMENSION DATA(16,16), GNEW(16,16), AGNEW(16,16),
1 AGRW(16,16), ADSTD(16,16)
  READ(5,98) (KI,L,C,LP,LA)
  K = KI - 1.
  AJ = FLOAT(KI) * FLOAT(K) / 2.
  AK = AJ - 1.
  WRITE(6,125)
  SMEAN = 0
  DO 4 I = 1,K
  N = KI - I
  DO 4 J = 1,N
  AGNEW(I,J) = 0
  AGRW(I,J) = 0
  ADSTD(I,J) = 0
4 CONTINUE
5 DO 29 LI = 1,L
  READ(5,99) (KN)
  IF (LP) 8, 8, 7
7 WRITE(7,160) KN
8 WRITE(6,160) KN
  SUM = 0
  SUMX = 0
  DO 10 I = 1,K
  N = KI - I
10 READ(5,100) (DATA(I,J),J=1,N)
  DO 11 I = 1,K
  N = KI - I
  DO 11 J = 1,N
  AGRW(I,J) = AGRW(I,J) + DATA(I,J)
  SUM = SUM + DATA(I,J)
11 SUMX = SUMX + DATA(I,J) ** 2
  AMEAN = (SUM / AJ)
  SMEAN = SMEAN + AMEAN
  STD = ((SUMX - (SUM ** 2 / AJ)) / AK) ** .5
  WRITE(6,150) SUM,SUMX,AMEAN,STD
  DO 12 I = 1,K
  N = KI - I
  DO 12 J = 1,N
  AGNEW(I,J) = AGNEW(I,J) + ((DATA(I,J) - AMEAN) / STD)
  GNEW(I,J) = ((DATA(I,J) - AMEAN) / STD) + C
  ADSTD(I,J) = ADSTD(I,J) + GNEW(I,J)
12 CONTINUE
  DO 29 I = 1,K
  N = KI - I
24 IF (LP) 26, 26, 25
25 WRITE(7,300) (GNEW(I,J), J = 1,N)
26 WRITE(6,200) (GNEW(I,J), J = 1,N)
29 CONTINUE
  IF (LA) 80, 80, 30
30 SMEAN = SMEAN / FLOAT(L)
  WRITE(6,250) SMEAN
  IF (LP) 36, 36, 35
35 WRITE(7,170)
36 WRITE(6,170)
  DO 40 I = 1,K
  N = KI - I
  DO 40 J = 1,N
40 AGNEW(I,J) = (AGNEW(I,J) / FLOAT(L)) + C
  DO 50 I = 1,K
  N = KI - I
  IF (LP) 46,46,45
45 WRITE(7,300) (AGNEW(I,J),J=1,N)
46 WRITE(6,200) (AGNEW(I,J),J=1,N)
50 CONTINUE
  SUM = 0
  SUMX = 0
```



```

DO 55 I = 1,K
N = KI - I
DO 55 J = 1,N
AGRAW(I,J) = AGRAW(I,J) / FLOAT(L)
SUM = SUM + AGRAW(I,J)
55 SUMX = SUM + AGRAW(I,J) ** 2
AMEAN = SUM / AJ
STD = ((SUMX - (SUM ** 2 / AJ)) / AK) ** .5
WRITE(6,150) SUM, SUMX, AMEAN, STD
IF (LP) 58, 58, 57
57 WRITE(7,180)
58 WRITE(6,180)
DO 60 I = 1,K
N = KI - I
DO 60 J = 1,N
60 AGRAW(I,J) = ((AGRAW(I,J) - AMEAN) / STD) + AMEAN
DO 65 I = 1,K
N = KI - I
IF (LP) 63,63,62
62 WRITE(7,300) (AGRAW(I,J),J=1,N)
63 WRITE(6,200) (AGRAW(I,J),J=1,N)
65 CCNTINUE
SUM = 0
SUMX = 0
DO 70 I = 1,K
N = KI - I
DO 70 J = 1,N
ADSTD(I,J) = ADSTD(I,J) / FLOAT(L)
SUM = SUM + ADSTD(I,J)
70 SUMX = SUMX + ADSTD(I,J) ** 2
AMEAN = SUM / AJ
STD = ((SUMX - (SUM ** 2 / AJ)) / AK) ** .5
WRITE(6,150) SUM, SUMX, AMEAN, STD
IF (LP) 73, 73, 72
72 WRITE(7,190)
73 WRITE(6,190)
DO 75 I = 1,K
N = KI - I
DO 75 J = 1,N
75 ADSTD(I,J) = ((ADSTD(I,J) - AMEAN) / STD) + AMEAN
DO 80 I = 1,K
N = KI - I
IF (LP) 78,78,77
77 WRITE(7,300) (ADSTD(I,J),J=1,N)
78 WRITE(6,200) (ADSTD(I,J),J=1,N)
80 CONTINUE
98 FORMAT(I2, I3, F9.3, I2, I2)
99 FORMAT(I4)
100 FORMAT(16F2.0)
125 FORMAT('1', 5X, 'STANDARDIZED MATRICES FOR INDIVIDUAL
1RESPONDENTS FOLLOW')
150 FORMAT('0', 'SUMMED CELLS =', F18.3,
1' SUM OF CELLS SQUARED =', F18.3, 'MATRIX MEAN = ',
1F8.3, 'STANDARD DEVIATION = ', F8.3)
160 FORMAT('0', 5X, 'STANDARD SCORES ADJUSTED TO CONSTANT,
1RESPONDENT NO. ', I4)
170 FORMAT(5X, 'MEAN OF AGGREGATED STD SCORES, ADJUSTED TO
1CONSTANT')
180 FORMAT(5X, 'AGGREGATED RAW SCORES STANDARDIZED &
1ADJUSTED TO MEAN')
190 FORMAT(5X, 'AGGREGATED ADJUSTED STD SCORES,
1STANDARDIZED & ADJUSTED TO MEAN')
200 FORMAT(5X, 16F8.3)
250 FORMAT('1', 5X, 'AGGREGATE MEAN IS', F12.3)
300 FORMAT(10F8.3)
399 FORMAT(2F8.3)
400 STOP
END

```

```

//GO.FT07F001 DD SYSOUT=B
//GO.SYSIN DD *
INSERT PARAMETER CARD HERE.
INSERT HEADER CARD HERE.

```


DATA DECK FOR FIRST RESPONDENT HERE.
HEADER CARD FOR NEXT RESPONDENT HERE.
DATA DECK FOR NEXT RESPONDENT HERE.
CONTINUE WITH HEADER CARDS AND DATA DECKS AS REQUIRED.
"SLANT-STAR" END-JOB CARD HERE.

*** IF THE USER DESIRES TO OBTAIN PUNCHED CARD OUTPUT OF
AGGREGATE MATRICES ONLY, INSTRUCTIONS 24 & 25 SHOULD BE
REMOVED FROM THE DECK AND THE PROGRAM RUN AS USUAL. THE
INDIVIDUAL MATRICES WITH THE RESPONDENT'S NUMBERS WILL BE
PRINTED AS A RECORD OF THE RESPONDENTS INCLUDED IN THE
AGGREGATION FOR THAT RUN. ***

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